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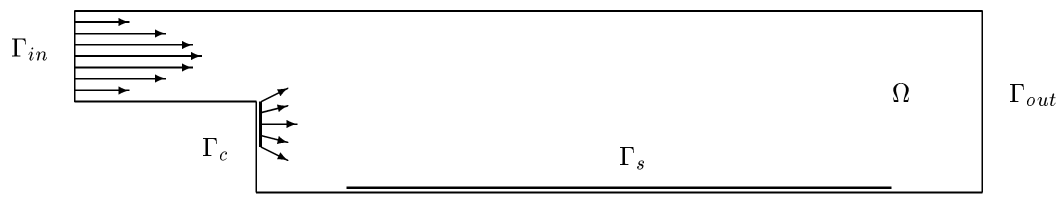
Some Aspects for Instantantenous Boundary Control of Backward-Facing Step Flow

In the present article a boundary control/boundary observation situation for time-dependent incompressible Navier-Stokes flow at low Reynolds number over a backward-facing step is studied. The control action aims at reducing the recirculation behind the step. Particular attention is given to the influence of the location of the control boundary on the efficiency of the control. Numerical examples are presented.

1. Introduction

Consider the flow of a fluid over the two-dimensional backward-facing step:



The fluid is supposed to be governed by the incompressible Navier-Stokes equations

$$\begin{aligned} \tfrac{\partial v}{\partial t} + (\nabla v)v - \tfrac{1}{Re}\Delta v + (\nabla p)^T &= 0 \quad \text{on } \Omega \times [0,T] \\ \operatorname{div} v &= 0 \quad \text{on } \Omega \times [0,T] \end{aligned} \tag{1}$$

with given initial and boundary conditions. Here, a parabolic inflow profile on Γ_{in} and natural boundary conditions $\frac{1}{Re}\partial_n v = p\, n$ on Γ_{out} (as introduced in [3]) are assumed. Control is exerted by means of the boundary values on Γ_c which is either the upper or lower part of the back of the step. The remaining part of the boundary is subject to a no-slip condition.

For moderate Reynolds numbers $Re = \frac{Uh}{\nu}$ (where U is the bulk velocity of the inflow, h is the step height and ν is the viscosity of the fluid) the flow field starting from rest at $t = 0$ becomes stationary and a recirculation zone evolves behind the step. The objective function (where ∂_n denotes the derivative in the direction of the outer normal)

$$J(\phi) = \int_0^T \left[\int_{\Gamma_s} \frac{1}{2}\partial_n v(|\partial_n v| + \partial_n v)\, d\Gamma_s + \frac{\gamma}{2}\int_{\Gamma_c} |\phi|^2\, d\Gamma_c \right] dt \tag{2}$$

aims at reducing the recirculation by penalizing backflow near the sensor boundary Γ_s while also measuring the control effort.

2. Instantaneous Control

The concept of instantaneous control for this very setting has been introduced in [1]. In short, it applies the time scheme (with time step τ)

$$\frac{v^{n+1} - v^n}{\tau} - \frac{1}{Re}\Delta v^{n+1} + (\nabla v^n)v^n + (\nabla p^{n+1})^T = 0 \tag{3}$$

to the time-dependent equation (1) to obtain a sequence of optimal control problems for the stationary Quasi-Stokes equations. Instead of the objective functional (2) its stationary counterpart

$$J(\phi) = \int_{\Gamma_s} \frac{1}{2}\partial_n v(|\partial_n v| + \partial_n v)\, d\Gamma_s + \frac{\gamma}{2}\int_{\Gamma_c} |\phi|^2\, d\Gamma_c \tag{4}$$

is used. The stationary control problems are not solved exactly, but only one gradient step is performed for each problem. This suboptimal approach has proven very efficient in reducing computational cost while achieving reasonable results.

3. Numerical Results

All computations were done on a domain Ω with vertices (0,0), (2,0), (2,-1), (10,-1), (10,1), and (0,1), with an inflow profile of $v = (10\,y\,(1-y), 0)$. The sensor boundary Γ_s was given by $[4,9] \times \{-1\}$. The control boundary Γ_c was either the upper half or the lower half of the back side of the step. The control action was computed until $T = 30$ with time step $\tau = 0.1$. Beforehand, a simulation with no control had been run with the fluid starting from rest in order to give the recirculation effect time to evolve completely. The Reynolds number and the control penalty parameter were chosen to be $Re = 300$ and $\gamma = 0.01$. At each time level, the initial choice for the control was $\phi^0 = 0$. Computing time amounted to about 5.5 hours for each of the configurations on a recent PC.

All flow problems were solved using the finite element multi-grid code FEATFLOW [2] on a uniform mesh with 1249 vertices and 1152 elements. For the optimal control problems the gradient algorithm suggested in [1] was used. The following graph shows the evolution of control cost $\frac{\gamma}{2}\int_{\Gamma_c} |\phi|^2 \, d\Gamma_c$ and backflow $\int_{\Gamma_s} \frac{1}{2}\partial_n v(|\partial_n v| + \partial_n v) \, d\Gamma_s$ over time. The latter term is computed based on the prediction of the state (v^{n+1}, p^{n+1}) which comes from the linearization (3). The actual value is significantly larger.

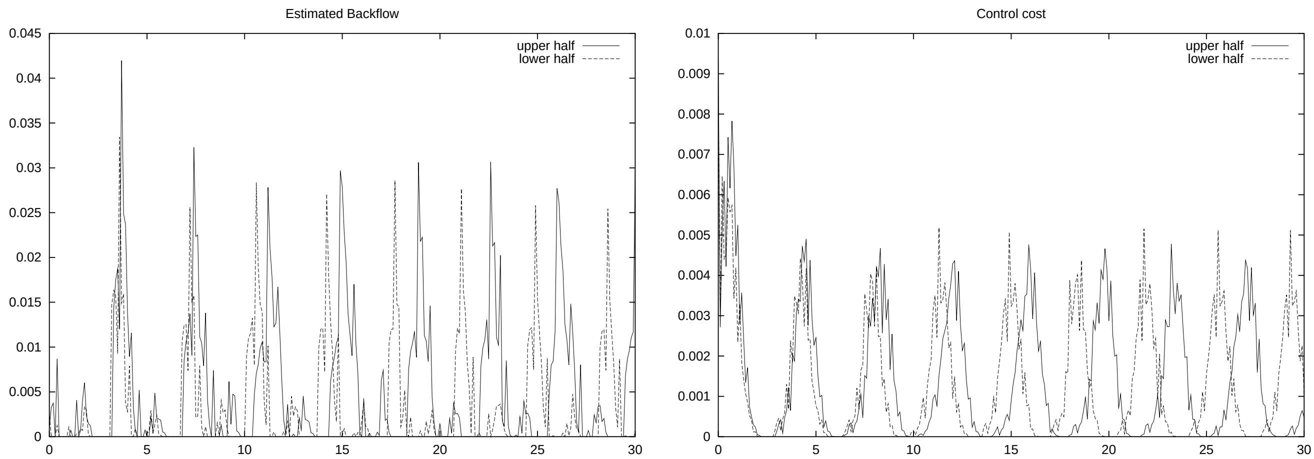


Figure 1: Estimated backflow (left) and control cost (right) over time for Γ_c being the upper half (solid) or lower half (dashed) of the back side of the step

As was observed in [1], the control action is quasi-periodic with the above mentioned choice of the sensor boundary Γ_s. It clearly shows that it does *not* make a difference in terms of control cost or control efficiency on which part of the boundary Γ_c the control acts. Nevertheless, one can observe that the fluid near the sensor boundary Γ_s reacts slightly faster to the control when the latter is exerted through the lower part of the back of the step. This gives rise to a shorter period in the control cost function. Movies and more examples can be found on my home page [4].

4. References

1 CHOI, H., HINZE, M., KUNISCH, K.: Instantaneous control of backward-facing-step flows. Applied Numerical Mathematics 31, 133-158 (1999)

2 TUREK, S., BECKER, C.: Featflow — Finite Element Software for the Incompressible Navier-Stokes Equations. User Manual Release 1.1. `http://www.featflow.de`

3 HEYWOOD, J., RANNACHER, R., TUREK, S.: Artificial boundaries and flux and pressure conditions for the incompressible Navier-Stokes equations. Int. J. Numer. Meth. Fluids 22, 325-352 (1996)

4 `http://www.staff.uni-bayreuth.de/~btmn07`

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