Robust Optimal Control: Introduction and Open Questions

Frank Schmidt[1]

[1]Chemnitz University of Technology, Faculty of Mathematics
Research Group Numerical Mathematics (Partial Differential Equations)

CHEMNITZ UNIVERSITY OF TECHNOLOGY

Chemnitzer Seminar zur Optimalsteuerung (Skiseminar)

March 7–14, 2009

Motivation

Given a optimization problem, depending on a parameter $p \in P_{\text{ad}}$ (typically compact)

$$\begin{aligned} \min_{u \in U} \quad & f_0(u,p) \\ \text{s.t.} \quad & f_i(u,p) \le 0 \qquad (i = 1, \dots, n) \end{aligned}$$

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Optimize the **worst-case** scenario

$$\begin{aligned} \min_{u \in U} \quad & \max_{p \in P_{\text{ad}}} f_0(u,p) \\ \text{s.t.} \quad & f_i(u,p) \leq 0 \quad \text{for all } p \in P_{\text{ad}} \qquad (i = 1, \ldots, n) \end{aligned}$$

Kinds of Robustness

Approximation problem $(A_0 + p\,A_1)\,u \approx b$

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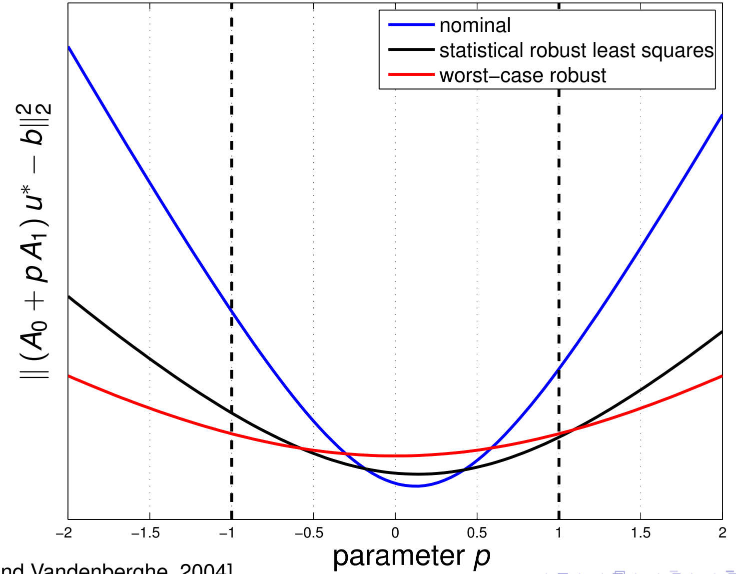
- **Worst-case Robust Problem**

$$\min_{u \in \mathbb{R}^n} \max_{p \in [-1,1]} \|\,(A_0 + p\,A_1)\,u - b\|_2^2$$

Comparison of Nominal, Stochastic and Worst-case



[Boyd and Vandenberghe, 2004]

Example: Chebyshev-Approximation

Let a function $g : \mathbb{R}^n \rightarrow \mathbb{R}$ and a set $\{\phi_1, \phi_2, \ldots, \phi_k\}$ of linearly independent polynomials be given.

Find $(u_1, u_2, \ldots, u_k)$ such that

$$\max_{x \in \Omega \subset \mathbb{R}^n} \left| \sum_{i=1}^{k} u_i \, \phi_i(x) - g(x) \right|$$

is minimal.

$x \mathrel{\widehat{=}}$ parameter p

Maximum Formulation

The **general robust** (worst-case) problem

$$\min_{u \in U} \quad \max_{p \in P_{\text{ad}}} f_0(u,p)$$

$$\text{s.t.} \quad f_i(u,p) \leq 0 \quad \text{for all } p \in P_{\text{ad}} \qquad (i = 1, \ldots, n)$$

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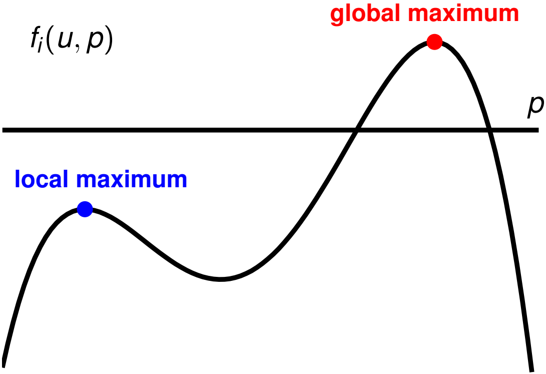
is equivalent to the **maximum formulation**

$$\begin{aligned} \min_{u\in U} \quad & \max_{p\in P_{\text{ad}}} f_0(u,p) \\ \text{s.t.} \quad & \max_{p\in P_{\text{ad}}} f_i(u,p) \leq 0 \qquad (i = 1,\ldots,n) \end{aligned} \tag{MF}$$

Problem of Feasibility

In general it is hard to determine whether u is feasible or not.



Numerical algorithms usually find **local extrema.**

Epigraph Formulation

The problem (MF) is equivalent to (**epigraph formulation**)

$$\min_{(u,\alpha)\in U\times\mathbb{R}} \quad \alpha$$

$$\text{s.t.} \quad \max_{p\in P_{\text{ad}}} f_0(u,p) \le \alpha$$

$$\max_{p\in P_{\text{ad}}} f_i(u,p) \le 0 \qquad (i = 1,\dots,n)$$

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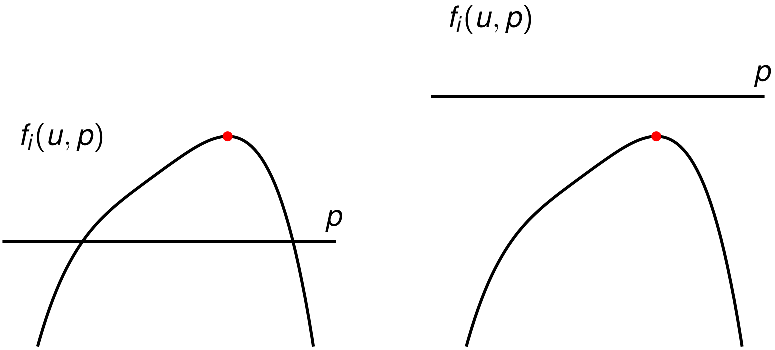
- Φ_i is **convex** if f_i is convex w.r.t. u
- Φ_i is **directionally differentiable** if $\frac{\partial f_i}{\partial u}$ exists and is continuous

Concave Problem

Suppose that f_i is concave w.r.t. p



local maximum = global maximum

Inner Optimality System ($\max_{p \in P_{ad}} f(u, p)$)

Assume

- $f(u, p)$ is concave w.r.t. p
- $P_{ad} = [0, 1]^m$
- some constraint qualification

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Then p solves $\max_{p \in P_{ad}} f(u,p)$ **iff** there exist Lagrange multipliers $\lambda_l,\ \lambda_u \in \mathbb{R}^m$ that fulfill the optimality system

$$\begin{aligned} \nabla_p f(u,p) + \lambda_l - \lambda_u &= 0 \\ p \geq 0, \quad \lambda_l \geq 0, \quad p^\top \lambda_l &= 0 \\ p \leq 1, \quad \lambda_u \geq 0, \quad (p-1)^\top \lambda_u &= 0 \end{aligned}$$

$$\min_{u,\alpha} \quad \alpha \qquad \text{s.t.} \quad \max_{p \in P_{\text{ad}}} f(u,p) \leq 0$$

MPCC

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$$\Longleftrightarrow$$

$$\min_{u,\alpha,p} \quad \alpha \qquad \text{s.t.} \quad f(u,p) \le 0$$

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$$\begin{aligned}
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&\text{s.t.} && f(u,p) && \le 0 \\
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& && p \ge 0, \quad \lambda_l \ge 0, \quad p^\top \lambda_l && = 0 \\
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\end{aligned}$$

Convexification

Use a **concave overestimator** $\tilde{f}(u,p) \geq f(u,p)$ instead of f.

$$\Longrightarrow \max_{p \in P_{\text{ad}}} f(u,p) \leq \max_{p \in P_{\text{ad}}} \tilde{f}(u,p) \overset{?!}{\leq} 0$$

If u is feasible w.r.t. $\tilde{f}$ then it is feasible w.r.t. f.

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$\tilde{f}_\gamma(u,p) := f(u,p) + \frac{\gamma}{2}\, p^\top (1-p)$ is such a overestimator.

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- $\tilde{f}_\gamma(u,\cdot)$ is concave if $\gamma \geq \max_{p \in P_{\text{ad}}=[0,1]^m} \|D_p^2 f(u,p)\|_2$
- $\tilde{f}_\gamma(u,p) = f(u,p) \quad \forall p \in \partial P_{\text{ad}} = \partial[0,\,1]^m$

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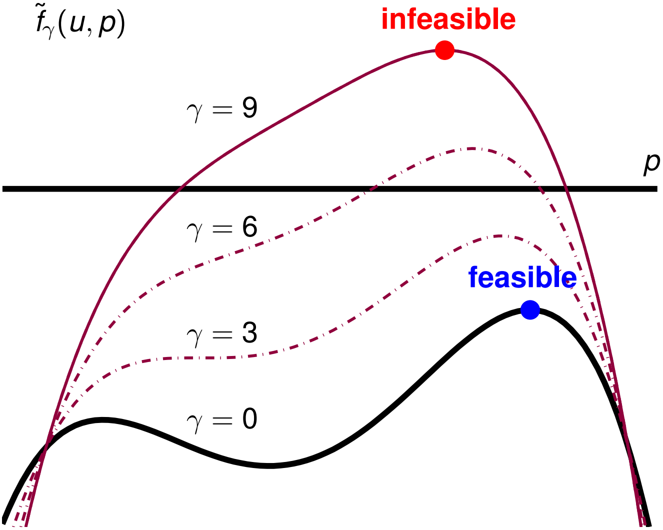
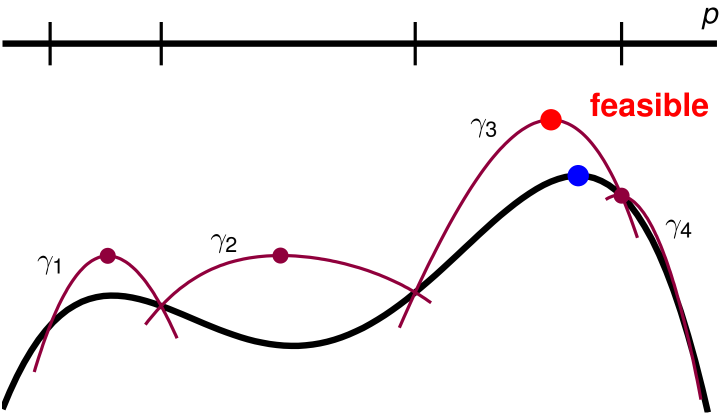


[Floudas and Stein, 2007]

Piecewise Convexification



Adaptive convexification is possible!

[Floudas and Stein, 2007]

Outline

Example: Perturbed Controls

$$\min_{u \in U_{\text{ad}}} \quad \max_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u + p\|^2_{L^2(\Omega)}$$

$$\text{s.t.} \qquad \begin{aligned} -\triangle y + y &= u + p && \text{in } \Omega \\ \partial_\nu y &= 0 && \text{on } \Gamma \end{aligned}$$

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Theorem

There exists a worst-case optimal control $u^* \in U_{\text{ad}}$.

Example: Perturbed Controls (cont'd)

$$\min_{u \in U_{\text{ad}}} \Phi_0(u) \qquad \text{with } \Phi_0(u) = \max_{p \in P_{\text{ad}}} f_0(u,p)$$

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2. There exists a subsequence $u_{n'}$ such that $u_{n'} \rightharpoonup u^* \in U_{ad}$
3. $j = \lim_{n'} \Phi_0(u_{n'}) = \liminf_{n'} \Phi_0(u_{n'}) \geq \Phi_0(u^*) \Rightarrow \Phi_0(u^*) = j$

Example: Uncertain Coefficient (linear)

$$\min_{u \in U_{\text{ad}}} \quad \max_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}$$

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Question

Is there a worst-case optimal control $u^* \in U_{\text{ad}}$?

$$\min_{u \in U_{\text{ad}}} \Phi_0(u) \qquad \text{with } \Phi_0(u) = \max_{p \in P_{\text{ad}}} f_0(u, p)$$

Attempt of a Proof

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3. $\exists \{p_{n'}\} \subset P_{ad}$ such that $f_0(u_{n'}, p_{n'}) = \Phi_0(u_{n'})$

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3. $\exists \{p_{n'}\} \subset P_{ad}$ such that $f_0(u_{n'}, p_{n'}) = \Phi_0(u_{n'})$
4. There exists a subsequence $p_{n'}$ with $p_{n'} \to p^* \in P_{ad}$

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$$\min_{u \in U_{\text{ad}}} \Phi_0(u) \qquad \text{with } \Phi_0(u) = \max_{p \in P_{\text{ad}}} f_0(u,p)$$

Attempt of a Proof

1. $\exists \{u_n\} \subset U_{\text{ad}}$ such that $\Phi_0(u_n) \to j := \inf_{u \in U_{\text{ad}}} \Phi_0(u)$
2. There exists a subsequence $u_{n'}$ such that $u_{n'} \rightharpoonup u^* \in U_{\text{ad}}$
3. $\exists \{p_{n'}\} \subset P_{\text{ad}}$ such that $f_0(u_{n'}, p_{n'}) = \Phi_0(u_{n'})$
4. There exists a subsequence $p_{n'}$ with $p_{n'} \to p^* \in P_{\text{ad}}$
5. $j = \liminf_{n'} f_0(u_{n'}, p_{n'}) \geq \liminf_{n'} f_0(u_{n'}, p^*) \geq f_0(u^*, p^*) \stackrel{?}{=} \Phi_0(u^*)$

Outline

Roadmap

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4. Implementation

Finish

Thank You!