Hierarchical Tensors Approximation for Uncertainty Quantification

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Hierarchical Tucker tensor format (HT - Hackbusch tensors) and Tensor Trains (TT- Tyrtyshnikov tensors, I.Oseledets) have been introduced recently for low rank tensor product approximation. Hierarchical tensor decompositions are based on sub space approximation by extending the Tucker decomposition into a multi-level framework. Therefore they inherit favorable properties of Tucker tensors, e.g they offer a stable and robust approximation, but still enabling low order scaling with respect to the dimensions. For many high dimensional problems, hard to be handled so far, this approach may offer a novel strategy to circumvent the curse of dimensionality.

For uncertainty quantification we cast the original boundary value problem, with uncertain coefficients problem into a high dimensional parametric boundary value problem, discretized by Galerkin method. The high dimensional problem is cast into an optimization problems, constraint by the restriction to tensors of prescribed ranks $\mathbf{r}$. This problem could be solved by optimization on manifolds, or more simply by alternating least squares. Since the norm of the underlying energy-space is a cross norm preconditioning is required only for the spatial part and e.g. performed by standard multi grid approaches, e.g BPX. Of Importance is, that this leads to a modification of the orthogonality of the used component tensors.

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