A Posteriori Error Control in Stochastic FEM and MLMC

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While a posteriori error estimation is a mature field in computational PDE, in particular FEM, there are only few adaptive methods for PDE with stochastic data as yet. We consider an elliptic model problem with dependence on a countable infinite set of independent uniform random variables. It is reformulated as parametric problem and discretised with the Stochastic Galerkin FEM in Legendre polynomial chaos. The introduction of a residual based reliable error estimator enables the formulation of an adaptive algorithm. The splitting into approximation and stochastic tail residuals leads to a refinement of the physical FEM space and of the anisotropic polynomial chaos simultaneously. In particular, the number of stochastic dimensions is adjusted in a problem-dependent way. Moreover, the algorithm can be shown to result in a convergent series of Galerkin discretisations. An extension of recent higher-order equilibration techniques facilitates the computation of guaranteed error estimators which do not involve unknown constants. Several benchmark problems illustrate the performance of the devised adaptive algorithm. Additionally, we give an outlook on how goal-oriented a posteriori error estimation can be included in FE-MLMC computations.

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