A Robust Petrov-Galerkin Discretisation of Convection-Diffusion Equations

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A Petrov-Galerkin finite element discretization is presented of an ultra-weak variational formulation of the convection-diffusion equation in mixed form. Here, an optimal test space is introduced to get the best approximation to the solution from the trial space.

To arrive at an implementable method, the truly optimal test space has to be replaced by its projection onto a

finite dimensional test search space. To prevent that this latter space has to be taken increasingly large for vanishing diffusion, a formulation is constructed that is well-posed in the limit case of a pure transport problem.

Numerical experiments show approximations that are very close to the best approximations, uniformly in the size of the diffusion term.

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