Guaranteed Energy Error Estimators for a Modified Crouzeix-Raviart Stokes Element

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This paper provides guaranteed upper energy error bounds for a modified lowest-order nonconforming Crouzeix-Raviart finite element method for the Stokes equations. The modification delivers a more robust Crouzeix-Raviart finite element method, that allows an optimal pressure-independent energy error estimate. In case of a right-hand side with large irrotational component in the sense of the Helmholtz decomposition, the modified element can lead to errors that are smaller by several magnitudes. Former guaranteed upper bounds for the unmodified Crouzeix-Raviart element are still applicable with small modifications, but result sometimes in a huge overestimation. To be efficient with respect to the modified solution, guaranteed upper bounds must approximate the divergence-free part of the Helmholtz decomposition of the right-hand side. Some designs are compared and verified by numerical benchmark examples. They show that guaranteed error control for the modified element is possible and almost as sharp as for the unmodified element.

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