Tensor numerical methods for multi-dimensional PDEs

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Most common in computational practice separable representations of functions and operators combine the canonical, Tucker, tensor train (TT) and the quantized-TT (QTT) formats. The QTT tensor format, invented in 2009 [1], makes it possible to represent the multivariate functions, operators and dynamical systems in quantized tensor spaces with logarithmic complexity scaling in the size of the full tensor. QTT approximation applies to the quantized images, obtained by reshaping up to the 'indivisible' mode size '2', that represents a *quant* of information (reminiscent quantum bits, i.e. qubits in quantum computations). The numerical efficiency is justified by the remarkable QTT-approximation properties proven for the wide class of functions and operators. We focus on the main approximation and complexity results in the quantized tensor formats recently applied to the solution of d-dimensional elliptic and parabolic equations [2]. Numerical tests indicate the logarithmic computational complexity of the QTT-tensor method in application to the parametric elliptic PDEs, computational quantum chemistry and to the multi-dimensional dynamics.

References:

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[2] B.N. Khoromskij, *Introduction to Tensor Numerical Methods in Scientific Computing.* Lecture Notes, Preprint 06-2011, University of Zuerich, Institute of Mathematics, 2011, pp. 1 - 238. http://www.math.uzh.ch/fileadmin/math/preprints/06_11.pdf.

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