Schwarz type solvers for hp-FEM discretizations of mixed problems

Sven Beuchler[1] Martin Purrucker[2]

The Stokes problem and linear elasticity problems can be viewed as a mixed variational formulation. These formulations are discretized by means of the hp-version of the finite element method. The system of linear algebraic equations is solved by the preconditioned Bramble-Pasciak conjugate gradient method. The development of an efficient preconditioner requires three ingredients, a preconditioner related to the components of the velocity modes, a preconditioner for the Schur complement related to the components of the pressure modes and the discretization by a stable finite element pair which satisfies the discrete inf-sup condition. The last condition is also important in order to obtain a stable discretization scheme. The preconditioner for the velocity modes is adapted from fast hp-FEM preconditioners for elliptic problems. Moreover, we will prove that the preconditioner for the Schur complement can be chosen as a diagonal matrix if the pressure is discretized by discontiuous finite elements. We will prove that the system of linear algebraic equations can be solved in almost optimal complexity if the $Q_k - P_{k-1,disc}$ element is used. This yields to quasioptimal hp-FEM solvers for the Stokes problems and linear elasticity problems. The latter are robust with respect to the contraction ratio ν. The efficiency of the presented solver is shown in several numerical examples.

[1] Institute for Numerical Simulation, University of Bonn, Wegelerstrasse 6, 53115 Bonn, Germany, beuchler@ins.uni-bonn.de

[2] Institute for Numerical Simulation, University of Bonn, Wegelerstrasse 6, 53115 Bonn, Germany, purrucker@ins.uni-bonn.de