Higher order variational time discretizations for nonlinear systems of ordinary differential equations

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We discuss different time discretizations of variational type applied to a nonlinear system of ordinary differential equations which is generated by a semi-discretization in space of a given nonlinear parabolic partial differential equation like, for instance, the non-stationary Burgers equation.

Among these methods we compare the known continuous Galerkin-Petrov and the discontinuous Galerkin method with time polynomial ansatz functions of order k (cGP(k)- and dG(k)-method) with respect to accuracy, stability and computational costs. Moreover, we propose two new extended methods (cGP-C1($k+1$)- and dG-C0($k+1$)-method) which have on the one hand a one higher degree of ansatz functions and accuracy, the same stability properties and on the other hand the same computational costs as the original methods.

We present optimal error estimates and the close relationship between the original and extended methods which prove as a byproduct the super-convergence of the original methods cGP(2) and dG(1) in the endpoints of the discrete time intervals. Finally, we present first numerical results for the non-stationary Burgers equation in one space dimension.

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