Some finite element approaches for contact/obstacle problems

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The first part of the talk deals with dual formulations for unilateral contact problems with Coulomb friction. Starting from the complementary energy minimization problem, Lagrangian multipliers are introduced to include the governing equation, the symmetry of the stress tensor as well as the boundary conditions on the Neumann and contact boundary. Since the functional arising from the friction part is nondifferentiable an additional Lagrangian multiplier is introduced. This procedure yields a dual-dual formulation of a two-fold saddle point structure. Two different Inf-Sup conditions are introduced to ensure existence of a solution. The system is solved with a nested Uzawa algorithm.

In the second part of the talk a mixed hp-time discontinuous Galerkin method for elasto-dynamic contact problem with friction is considered. The contact conditions are resolved by a biorthogonal Lagrange multiplier and are component-wise decoupled. On the one hand the arising problem can be solved by an Uzawa algorithm in conjunction with a block-diagonalization of the global system matrix. On the other hand the decoupled contact conditions can be represented by the problem of finding the root of a non-linear complementary function. This non-linear problem can in turn be solved efficiently by a semi-smooth Newton method. The second method can also be applied to parabolic obstacle problems, e.g. pricing American put options.

In all cases numerical experiments are given demonstrating the strengths and limitations of the approaches.

The talk is based on a joint work with M. Andres and L. Banz.

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