A non-standard finite element method for convection-diffusion-reaction problems on polyhedral meshes

Clemens Hofreither[1]   Ulrich Langer[2]   Clemens Pechstein[3]

We present a new non-standard finite element method based on element-local boundary integral operators that permits polyhedral element shapes as well as meshes with hanging nodes. The method employs elementwise PDE-harmonic trial functions and can thus be interpreted as a local Trefftz method.

The construction principle requires the explicit knowledge of a fundamental solution of the partial differential operator, but only locally, i.e., in every polyhedral element. Fundamental solutions are known for general elliptic PDEs of diffusion-convection-reaction type with constant coefficients. This allows us to solve such PDEs with elementwise constant coefficients.

In this talk, we apply the method to convection-diffusion problems, in particular convection-dominated problems. We show that the PDE-harmonic trial functions lead to improved stability over standard piecewise linear ones and discuss possible further stabilization approaches.

[1] Johannes Kepler Universität, DK Computational Mathematics, Linz, Austria,
    `clemens.hofreither@dk-compmath.jku.at`

[2] Institute of Computational Mathematics, JKU Linz, Austria,

[3] Institute of Computational Mathematics, JKU Linz, Austria,