Preconditioned conjugate gradient method for optimal control problems with control and state constraints

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We consider saddle point problems arising as (linearized) optimality conditions in elliptic optimal control problems. In the spirit of Bramble and Pasciak, it turns out that the preconditioned systems are symmetric positive definite with respect to a suitable scalar product. We extend previous work of Schöberl and Zulehner by considering problems with control and state constraints. It stands out as a particular feature of this approach that only those matrices which represent the inner products need to be preconditioned, e.g., by multigrid cycles. Thus the preconditioner becomes independent of the underlying forward and adjoint differential equations. Numerical examples in 2D and 3D are given which illustrate the performance of the method.

References:

[1] J. Schöberl, W. Zulehner, Symmetric Indefinite Preconditioners for Saddle Point Problems with Applications to PDE-Constrained Optimization, SIAM Journal of Matrix Analysis and Applications 29(3), p.752–773, 2007

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