Recent progress in the analysis of the convergence of FEM for Maxwell eigenvalue problems

Martin Costabel[1]

Finite element approximation of the Maxwell eigenvalue problem can notoriously go wrong, even if the standard "stability plus consistency" conditions are satisfied. The reason is that - depending on the choice of the variational formulation - one has to deal with a problem with a non-compact resolvent, or with a non-standard eigenvalue problem for a mixed formulation.

Recently, D.N. Arnold, R. Falk and R. Winther have shown a general framework in which the spectrally correct approximation of the Maxwell eigenvalue problem is a reward for the obedience of the finite element spaces to some algebraic structure, in the presence of uniform estimates of compatible interpolation operators in the appropriate norms (keywords "discrete subcomplexes of the de Rham complex" and "uniformly bounded cochain projectors"). These arguments cover the spectrally correct convergence for the h version FEM using edge elements of arbitrary order on simplicial meshes.

For the p and hp versions of the edge element method, the available known interpolants do not quite fit into this scheme. In particular, it is hard to prove L^2 interpolation error estimates uniformly in p. A recently found tool, the regularized Poincaré integral operator, can help here. This tool answers a variety of questions in the regularity theory of vector analysis on Lipschitz domains. In the analysis of computational electromagnetics, it can be used to complete the proof of the discrete compactness property of the p version edge element methods on various 2 and 3 dimensional meshes, thereby showing spectrally correct convergence of these methods. These results have been obtained in joint work with D. Boffi (Pavia), M. Dauge (Rennes), L. Demkowicz (Austin), R. Hiptmair (Zürich), A. McIntosh (Canberra).

[1] IRMAR, Université de Rennes 1, France,
martin.costabel@univ-rennes1.fr