Numerical challenges in modeling Stokes for planetary mantle convection

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Mantle convection simulation is challenging because it needs huge parallel computers and adaptivity to control the strongly varying parameters. In the case of planetary core simulations also the Maxwell equations are involved. In this talk planetary convection is investigated from a numerical point of view, theoretical analysis as well as practical tests are performed. The stability criteria for the numerical formulation of the physical model will be made clear using the well known simulation code TERRA as an example. For the incompressible case and the TERRA specific treatment of the an-elastic approximation, two inf-sup stable grid modifications can be found, which are both compatible with hanging nodes. For the $Q_{1_h}Q_{1_{2h}}$ element pair a simple numeric test is introduced to prove the stability for any given (TERRA-)grid. For the $Q_{1_h}P^{disc}_{1_{2h}}$ element pair and 1-regular refinements with hanging nodes an existing general proof can be adopted. The influence of the slip boundary condition is known to be destabilizing. For the incompressible case a cure can be adopted from the literature. but the general case is still not clear. The necessary conditions for the expansion of the stability results to the an-elastic approximation will be pointed out.
Additionally a small numerical framework is presented in order to measure the effect of different numerical approaches to improve the handling of strongly varying viscosity. The framework is applied to investigate how block smoothers with different block sizes, combination of different block smoothers, different prolongation schemes and semi coarsening influence the multi-grid performance.
The talk is intended to raise interest of the mathematical experts for the numerical challenges of this physical model.

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