Stabilized finite element methods for nonstationary and nonlinear convection-diffusion-reaction equations

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Fluid flow problems with simultaneous reactive multicomponent transport of chemical species arise in many technical and environmental applications. The numerical solution of nonstationary convection-diffusion-reaction models is a challenge when convection is dominant and small layers of the concentrations arise. Here, the numerical approximation of such systems with SUPG stabilized higher order finite element methods and additional shock-capturing stabilization is studied theoretically and numerically. An anisotropic nonlinear variant of the shock-capturing methods is used.

As a model problem of our analysis we consider solving the scalar quasilinear convection-diffusion-reaction equation

$$\alpha u + \vec{b} \cdot \nabla u - \nabla \cdot (a \nabla u) + r(u) = f \text{ in } \Omega\,,$$
$$u = 0 \text{ on } \partial\Omega\,.$$

Rigorous error estimates are given for the numerical approximation scheme within a *hp* finite element framework and discussed in detail. In particular the design of the stabilization parameter is studied carefully.

Numerical results for transport systems of different complexities are presented. The results illustrate that shock-capturing in combination with a higher order finite element method is efficient to further reduce spurious oscillations in crosswind directions and avoid negative concentrations. This is of importance for coupled systems of equations in which inaccuracies in one concentration affect all other ones.

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