Local projection stabilisation on layer-adapted meshes for convection-diffusion problems with characteristic layers

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We consider singularly perturbed convection-diffusion problems on the unit square where the solution u exhibits exponential and characteristic layers. In order to stabilise the discretisation, layer-adapted meshes and the local projection method are applied.

Using bilinears, the error between the solution u and the finite element solution u^N converges with first order while the error between u^N and the bilinear interpolant $I^N u$ of the solution u shows second order convergence.

For enriched $\mathcal{Q}_p$ elements which already contain the space $\mathcal{P}_{p+1}$, the error between the solution u and the finite element solution u^N shows the convergence order $p+1$ in the ε-weighted energy norm. Furthermore, the error between u^N and a special interpolant $I^N u$ provides the convergence order $p+1$ in the local projection norm.

The theoretical results are confirmed by numerical results. The influence of the chosen enrichment will be discussed.

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