TU Chemnitz
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Nonlinear Optimization
Exercises 6

1. Considering the proof of Theorem VI.1 (inexact Newton), show that the $\{x_k\}$ converge to x^* in the $[\nabla^2 f(x^*)]^2$-norm at the same rate as the ∇f_k converge to zero; in particular, show for $q \geq 1$

$$\limsup_{k->\infty} \frac{\|\nabla f(x_{k+1})\|}{\|\nabla f(x_k)\|^q} = \alpha \in \mathbb{R} \qquad \Rightarrow \qquad \limsup_{k->\infty} \frac{\|x_{k+1}-x^*\|_{[\nabla^2 f(x^*)]^2}}{\|x_k - x^*\|^q_{[\nabla^2 f(x^*)]^2}} = \alpha.$$

2. Show that in the Line Search Newton Conjugate Gradients method the step direction p_k is always a descent direction.

3. Within the setting of the proof of Theorem IX.13 show that the matrix

$$\begin{bmatrix} \nabla^2 f(x^*) - \sum \lambda_i^* \nabla^2 c_i(x^*) & -[\nabla c_1(x^*), \ldots, \nabla c_m(x^*)] \\ [\nabla c_1(x^*), \ldots, \nabla c_m(x^*)]^T & 0 \end{bmatrix}$$

is nonsingular.

4. Consider the following quadratic program in $\mathbb{R}^2$

$$\begin{array}{ll} \min & \frac{1}{2} x^T A x - bx \\ \text{s.t.} & x_1 + 1 \geq 0 \\ & -x_1 + 1 \geq 0 \\ & x_2 + 1 \geq 0 \\ & -x_2 + 1 \geq 0 \ . \end{array}$$

Determine the optimal solutions, their Lagrange multipliers and the (strongly/weakly) active sets for the cases

(a) $A = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 14 \\ 7 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

Try to determine in each case the optimal solution in dependence of a small displacement $d \in \mathbb{R}^4$ of the right hand side.