TU Chemnitz
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Chemnitz, May 10, 2004
due on: May 19,2004

Nonlinear Optimization
Exercises 5

1. Implement the dogleg trust region method in Matlab. As before, the function may be given by any routine of the form `[f,g,h]=functionname(x)` where `f`= $f(x)$, `g`=$\nabla f(x)$ and `h`= $\nabla^2 f(x)$ (all vectors are column vectors), and the optimization routine must be callable by `[xsol]=dogleg(@functionname,xstart)`. Send the source code of your m-file as pure ASCII text to `helmberg@mathematik.tu-chemnitz.de` with the subject "`NLO04, dogleg, your name`" till , May 19 (2004), 14:00.

2. Suppose the values of a function f contain roundoff errors u. Show that a suitable value for the perturbation ϵ in the central-differnce formula is approximately $\epsilon = u^{1/3}$, and that the accuracy achievable by this formula is approximately $u^{2/3}$. (Use similar assumptions to the ones used in the lecture to derive the estimates of ϵ for the forward-difference formula.)

3. Suppose the Jacobian of a function $f : \mathbb{R}^8 \longrightarrow \mathbb{R}^6$ has the following structure:

x	x	x					
x		x	x				
	x	x				x	
		x	x	x			
					x	x	x
				x	x		x

   where each row corresponds to the gradient of one coordinate function of f, each cross represents a nonzero element, and zeros are represented by blank space. Construct the graph structure that can be used to determine the full Jacobian, and find a valid coloring scheme with three colors. Are three colors minimum and if so, why?

4. For the unitary operation $x_i = L(x_j)$ $(x \in \mathbb{R}^n)$ verify the differentiation formulae

$$\begin{aligned} D_p x_i &= L'(x_j)(D_p x_j) \\ D_{pq} x_i &= L''(x_j)(D_p x_j)(D_q x_j) + L'(x_j) D_{pq} x_j \end{aligned}$$

   where $D_p x_i$ and $D_{pq} x_i$ are defined as follows:

$$D_p x_i := (\nabla x_i)^T p = \sum_j \frac{\partial x_i}{\partial x_j} p_j, \qquad D_{pq} x_i := p^T (\nabla^2 x_i) q, \qquad p, q \in \mathbb{R}^n.$$