TU Chemnitz
C. Helmberg

Chemnitz, April 21, 2004

Nonlinear Optimization
Exercises 3

1. The *Coordinate Descent* or *Gauss-Seidel* Method is a particularly simple optimization algorithm that optimizes a function $f : \mathbb{R}^n \to \mathbb{R}$ by one coordinate at a time (all other coordinates are fixed at their current values), i.e. the search directions are $\pm e_1, \pm e_2, \ldots, \pm e_n, \pm e_1, \ldots$. Exhibit some examples where this is a very bad idea.

2. A popular technique to improve the convergence of heuristics as the one above is to do a steepest descent step every so many, say n, iterations. Does this technique of introducing so called *spacer steps* help in establishing convergence?

3. (Proof of Theorem II.7) Prove

   (a) $\|x_k + p_k - x^*\| \leq O(\|x_k - x^*\|^2) + o(\|p_k\|) \quad \Rightarrow \quad \|p_k\| = O(\|x_k - x^*\|)$

   (b) $\|x_k + p_k - x^*\| \leq o(\|x_k - x^*\|) \quad \Rightarrow \quad \|p_k\| = O(\|x_k - x^*\|)$ and $\|p_k - p_k^N\| = o(\|p_k\|)$

4. Implement a Matlab routine for a line search Newton Method with some globalization scheme (e.g. by ensuring the positive definiteness of the approximation of the Hessian) and a simple line search procedure (e.g. backtracking). The function may be given by any routine of the form `[f,g,h]=functionname(x)` where `f`$= f(x)$, `g`$=\nabla f(x)$ and `h`$= \nabla^2 f(x)$. The optimization routine must be callable by `[xsol]=lsnewton(@functionname,xstart)` (use `help feval` for evaluation of function parameters), where `xsol` should be a point with a gradient of small norm and `xstart` is a user specified starting point. Send the source code of your m-file as pure ASCII text to `helmberg@mathematik.tu-chemnitz.de` with the subject "`NLO04, lsnewton,` *`your name`*" till Wed, April 28 (2004), 14:00.