TU Chemnitz
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Nonlinear Optimization

Exercises 2

1. Derivative-free line search: A function $f : D \to \mathbb{R}$ with convex $D \subset \mathbb{R}^n$ is *strictly quasiconvex* if, for each $x_1, x_2 \in D$ with $f(x_1) \neq f(x_2)$,

$$f(\alpha x_1 + (1-\alpha)x_2) < \max\{f(x_1), f(x_2)\} \quad \forall \alpha \in (0,1).$$

Show: Let f be strictly quasiconvex on the interval $[a,b]$ and $a \leq \lambda < \mu \leq b$. If $f(\lambda) > f(\mu)$, then $f(\alpha) \geq f(\mu)\ \forall \alpha \in [a,\lambda]$. If $f(\lambda) \leq f(\mu)$ then $f(\alpha) \geq f(\lambda)\ \forall \alpha \in [\mu, b]$.

   (a) For the Fibonacci sequence $F_0 = F_1 = 1$, $F_{n+2} = F_{n+1} + F_n$ $(n \in \mathbb{N}_0)$ describe an algorithm that reduces in each iteration $0 \leq k \leq n-1$ the search interval $[a_k, b_k]$ by evaluating in

$$\lambda_k = a_k + \frac{F_{n-k-1}}{F_{n-k+1}}(b_k - a_k) \text{ and } \mu_k = a_k + \frac{F_{n-k}}{F_{n-k+1}}(b_k - a_k).$$

   (b) Do the same for the Golden Section Method with $\theta = \frac{\sqrt{5}-1}{2}$ and

$$\lambda_k = a_k + (1-\theta)(b_k - a_k), \quad \mu_k = a_k + \theta(b_k - a_k).$$

2. Line search by interpolation: Given a function $\Phi(\alpha) : \mathbb{R} \to \mathbb{R}$,

   (a) construct a quadratic function going through $\Phi(0)$ with slope $\Phi'(0)$ and through $\Phi(\alpha_0)$.

   (b) Build a cubic with the same properties using an additional point $\Phi(\alpha_1)$.

3. Design a smooth strictly convex function where Newton's method without line search does not converge.

4. Prove that for $0 < \lambda_1 \leq \cdots \leq \lambda_n$

$$\max_{\xi_i \geq 0, \sum \xi_i = 1} \left(\sum_{i=1}^n \xi_i \lambda_i\right)\left(\sum_{i=1}^n \xi_i \frac{1}{\lambda_i}\right) = \frac{1}{4}\frac{(\lambda_1 + \lambda_n)^2}{\lambda_1 \lambda_n}.$$

5. Consider the problem

$$\min f(x,y)$$

where $f(x,y) = \frac{cx^2+y^2}{2}$, $\quad (x,y) \in \mathbb{R}^2$, $\quad 0 < c < 1$. Show that , starting at $x_0 = (1, c)$, the steepest descent method with exact line search generates the points

$$x_k = (q^k, (-1)^k c q^k), \qquad k = 1, 2, 3, \ldots$$

where $q = \frac{1-c}{1+c}$. How many iterations are necessary to get to the minimum within an accuracy of 10^{-6} in case $c = 10^{-3}$.