Technische Universität Chemnitz
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Chemnitz, 23. January 2008
Deadline: 29. January 2008

Disposal until 14:00, in room Rh. 39/715!!

Introduction to Discrete Mathematics
Task 11

1. (6 scores)

   Sort the sequence 0,8,15,3,7,42,47,11 by Mergesort, Quicksort, and Heapsort and count the number of actually used comparisons.

2. (5 scores)

   Let $(p_1, \ldots, p_n)$ be a distribution and $q \geq 2$. Prove, that

   $$\overline{L}(p_1, \ldots, p_n) \leq \overline{L}(\frac{1}{n}, \ldots, \frac{1}{n})$$

   holds.

3. (5 scores)

   Write a program for a deterministic touring machine, which adds 1 to a given non-negative integer (given as binary number wiht most significant bit at position 1) while it does not accept an empty input.

   Test the program for the numbers five and three.

4. (4 scores)

   Prove the equivalency of the following statements for a given graph $G = (V, E)$ and a subset V' of its vertex set:

   (i) V' is a vertex cover of G (i.e., $\{u, v\} \in E \Rightarrow u \in V'$ oder $v \in V'$).
   (ii) $V - V'$ is an independent set of G.
   (iii) $V - V'$ induces a clique (complete subgraph) in the complementary graph $\overline{G}$.

   Conclude from it, that a polynomial algorithm (with respect to $|V|$ and $|E|$) for finding a minimal vertex cover also gives a polynomial algortihm for finding a maximal clique, and vice versa.