

Disposal until 14:00, in room Rh. 39/715!!

Introduction to Discrete Mathematics Task 10

1. (3 scores)

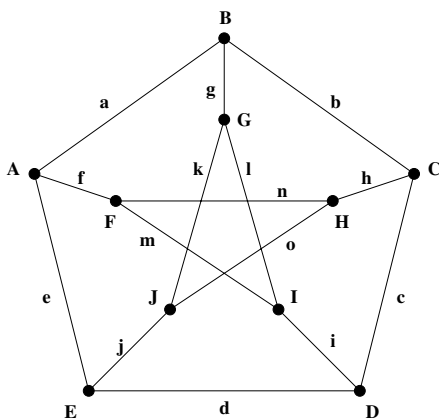
There are 6 computer programs. They should be run one after another on a mainframe (and then restarted). Each program needs its own resources, like a part of the central memory, a compiler, devices ... Furthermore the change from one program to another costs time. The following matrix $C = (c_{ij})$ contains all the times c_{ij} which are necessary for a change from one program P_i to another P_j .

Determine a sequence of programs, which uses a transmission time, which differ at most 50% from the optimal transmission time (that means the time from beginning of the first program until the next start of this program).

$$C = \begin{pmatrix} 0 & 18 & 17 & 23 & 12 & 19 \\ 18 & 0 & 26 & 31 & 20 & 30 \\ 17 & 26 & 0 & 16 & 11 & 9 \\ 23 & 31 & 16 & 0 & 17 & 19 \\ 12 & 20 & 11 & 17 & 0 & 14 \\ 19 & 30 & 9 & 19 & 14 & 0 \end{pmatrix}$$

2. (3 scores)

Show that the Petersen Graph is not hamiltonian.



3. (3 scores)

Let (c_{ij}) be a symmetrical cost matrix for the TSP with only non-negative entries, which satisfies the triangle inequality $c_{ik} \leq c_{ij} + c_{jk}$. Start with

an arbitrary vertex v and let $C_1 = \{v\}$. Let $C_k = \{u_1, \dots, u_k\}$ already be constructed. Find $u \notin C_k$ with minimal (cheapest) distance to C_k and introduce u before a vertex with minimum distance. Prove, that a tour constructed this way satisfies $c(T) \leq 2c(T_{opt})$.

4. (3 scores)

Solve the balancing problem, if it is known that the wrong coin is heavier.

5. (4 scores)

Let n and q be non negative integers and let $q \geq 2$. Prove that there is a complete (n, q) -tree if and only if $n - 1$ is a multiple of $q - 1$.

6. (4 scores)

Prove that every permutation a_1, a_2, \dots, a_n of the numbers $1, \dots, n$ can be ordered to $1, \dots, n$ by gradual exchanging neighbouring elements.

Example: $3123 \rightarrow 1324 \rightarrow 1234$

Find the minimum number of exchanges to order an arbitrary such permutation!