Technische Universität Chemnitz
S. Dienelt

Chemnitz, 05. December 2007
Deadline: 11. December 2007

Disposal until 14:00, in room Rh. 39/715!!

Introduction to Discrete Mathematics
Task 7

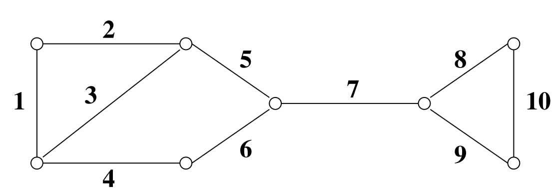
1. (4 scores) Let $T = \{t_1, \ldots, t_n\}$ be a set and $A_i \in 2^T$, $i = 1, \ldots, m$ ($A_i = A_j$ for different i, j allowed). If there is an injective map $\varphi : I \to \{1, \ldots, n\}$ with $t_{\varphi(i)} \in A_i$ (choice function) exists for a subfamily $\mathcal{A}_I = \{A_i : i \in I\}$ with $I \subseteq \{1, \ldots, m\}$ then $T_I = \{t_{\varphi(i)} : i \in I\}$ is called *transversal* or *system of distinct representatives* (SDR) of $\mathcal{A}_I$.

   Prove $(T, \mathcal{F} = \{T_I : \exists I \subseteq \{1, \ldots, m\} : T_I \text{ is transversal of } \mathcal{A}_I\})$ is a matroid (the so called *transversal matroid*).

   Hint: Construct a bipartite graph $(V = \mathcal{A}_{\{1,\ldots,m\}} \cup T, E = \{\{A_i, t_j\} : t_j \in A_i\})$; transversals are represented as matchings; if $X, Y \in \mathcal{F}$ with $|X| < |Y|$, then the union of the representing matchings contains an alternating path which enables to enlarge X.

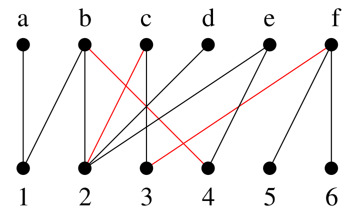
2. (4 scores) A (directed) path in a digraph $D = (V, A)$ is called hamilton path, if it meets all vertices of D. Reformulate the problem of finding a hamilton path in the fashion of a search for a maximum cardinality independent set in the intersection of three matroids.

3. (4 scores) Present the systems of cycles and cocycles of the graphic monoid of the following graph:



4. (2 scores) How can one apply shortest path algorithms for directed graphs also to undirected graphs with non negative edge weights? Is this possible for arbitrary edge weights, too?

5. (4 scores) Compute for the following bipartite graph a maximum matching using Edmonds' matroid intersection algorithm. Start with matching $X = \{2c, 3f, 4b\}$. Present in every iteration the graph D_x and the sets S_x, T_x.



Hint: Let $G = (V, E)$ be the given graph.

$\mathcal{F}_1$ includes all subsets $E' \subseteq E$ with: every vertex $v \in \{1, 2, 3, 4, 5, 6\}$ is incident to at most one edge in E'. $\mathcal{F}_2$ includes all subsets $E'' \subseteq E$ with: every vertex $v \in \{a, b, c, d, e, f\}$ is incident to at most one edge in E''. The intersection of the two matroids $(E, \mathcal{F}_1)$ and $(E, \mathcal{F}_2)$ is the independence system of all matchings of G.