Technische Universität Chemnitz
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Chemnitz, 28. November 2007
Deadline: 04. December 2007

Disposal until 14:00, in room Rh. 39/715!!

Introduction to Discrete Mathematics
Task 6

1. (4 scores) Describe the systems of circuits, bases and the independence system of the matrix matroid with respect to the columns of the following matrix:
$$A = \begin{pmatrix} 1 & -1 & 2 & 1 & -1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 & -1 \end{pmatrix}$$

2. (4 scores) Determine a minimal spanning tree for the following list of edges and edge weights of a graph on 14 vertices:

1 2 4.	2 3 5.	4 5 2.	6 7 2.	8 13 3.	10 11 5.
1 8 6.	2 14 3.	5 6 7.	6 13 6.	9 10 1.	10 12 3.
1 11 7.	3 4 1.	5 13 6.	7 8 6.	9 11 5.	11 12 6.
1 13 1.	3 14 1.	5 14 5.	7 9 3.	9 12 6.	13 14 2.

3. (4 scores) Let $G = (V, E)$ be an undirected graph with edge weights $w_e \in \mathbb{R}_+$ for $e \in E$. A *hamiltonian cycle* of G is a cycle covering V. The *Travelling Salesman Problem*, TSP is to determine a hamiltonian cycle of minimum weight. Give a formulation of this problem as a minimization problem over an independence system.

4. (2 + 2 scores) Prove the following:

   (a) Let $G = (V, E)$ be an undirected graph, $S \subseteq V$ be an independent set of G, and $k_s \in \mathbb{N}_0$ for all $s \in S$. Let $\delta_F(s)$ denote the set $\{e \in F : s \in e\}$, for $F \subseteq E$. Then $(E, \mathcal{F} = \{F \subseteq E : |\delta_F(s)| \leq k_s \ \forall s \in S\})$is a matroid.

   (b) Let $D = (V, A)$ be a directed graph, $S \subseteq V$, $k_s \in \mathbb{N}_0$ for all $s \in S$. Let $\delta^-_F(s)$ denote the set $\{a \in A : a = (u, s) \text{for a} u \in V\}$, for $F \subseteq A$. Then $(A, \mathcal{F} = \{F \subseteq A : |\delta^-_F(s)| \leq k_s \ \forall s \in S\})$ is a matroid.

Note: A set S of vertices of a graph G is called *independent* if G contains no edge connecting vertices of S.