Technische Universität Chemnitz
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Chemnitz, 14. November 2007
Deadline: 27. November 2007

**Disposal until 14:00, in room Rh. 39/715!!**

# Introduction to Discrete Mathematics
# Task 5

1. (2 scores) Suppose, there's an $n$-step-algorithm for input length $n$. Suppose further, step $i$ uses $i^9$ operations. Verify, that the running time of the algorithm is $O(n^{10})$.

2. (5 scores) Solve this problem by using an appropriate constructed graph:

   A wolf, a goat and a cabbage has to be carried over a river by a ferryman. The wolf would like to eat the goat, and the goat would like to eat the cabbage. Therefore neither wolf-goat nor goat-cabbage can be leave alone without supervision by the ferryman. The boat carries except the ferryman only one of the three. In which way he can carry all safely to the other shore?

3. (2 + 2 scores)

   (a) Show that each graph with $n$ nodes and $m$ edges has at least one node of degree $\geq \lceil \frac{2m}{n} \rceil$. For $x \in \mathbb{R}$, $\lceil x \rceil$ denotes the smallest integer, which is not smaller than $x$.

   (b) Prove, that any two longest paths of a connected graph share a common vertex!

4. (1 + 2 + 2 scores)

   (a) In which graphs the edge set contains bridges only?

   (b) Prove that an edge is a bridge if and only if it is not contained in a cycle!

   (c) Prove that a graph is bridgeless, if it contains vertices of even degree only.

5. (4 scores) A graph on 10 vertices is given by the following adjacency lists:

   | | | | |
   |---|---|---|---|
   | 1: 6,5,3,2 | 4: 2,3,5 | 7: 10 | 10: 7 |
   | 2: 1,3,4 | 5: 4,3,1,6 | 8: 9 | |
   | 3: 1,5,4,2 | 6: 1,9,5 | 9: 8,6 | |

   Run a BFS and a DFS algorithm starting with $v_0 = 1$ using the given order of adjacencies and figure out the resulting edge sets and vertex numberings!