Technische Universität Chemnitz
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Chemnitz, 07. November 2007
Deadline: 13. November 2007

**Disposal until 14:00, in room Rh. 39/715!!**

# Introduction to Discrete Mathematics
# Task 4

1. (2+2 scores) Proof the propositions:
   (a) Under the conditions of Polya's Theorem $|\mathcal{M}| = Z(G; r, r, r, \ldots, r)$ holds.
   (b) Under the conditions of Polya's Theorem, with $R = \{w,\ s\}$ $\sum_{k=0}^{n} a_k x^k = Z(G; 1+x, 1+x^2, \ldots, 1+x^n)$ holds. Thereby $a_k$ is the number of pattern, in which the colour white occurs exactly k-times.

2. (4 scores) How many patterns we get if we colour 6 edges of the cube white, but the other 6 edges black?

3. (1 + 1 + 1 scores) Show for the recursion $T(n) = aT(\lfloor n/b \rfloor) + f(n)$ with $f(n) = \Theta(n)$ that the following applies:
   (a) $T(n) = \Theta(n)$ for $1 \leq a < b$,
   (b) $T(n) = \Theta(n \lg n)$ for $1 < a = b$,
   (c) $T(n) = \Theta(n^{\log_b a})$ for $1 < b < a$.

4. (3 + 3 scores) Proof the equations:
   (a) $O(f(n))O(g(n)) = O(f(n)g(n))$,
   (b) If $f(n) > 0\ \forall n \in \mathbb{N}$, then $O(f(n)g(n)) = f(n)O(g(n))$ holds.