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Introduction to Discrete Mathematics
Exercise 9

1. Prove: Let E be a finite set and $E_1, \dots, E_k$ be non empty subsets of E satisfying $E_i \cap E_j = \emptyset$ for all $i \neq j$ and $\bigcup_{i=1}^{k} E_i = E$. If $b_1, \dots, b_k$ are non negative integers, then $(E, \mathcal{F} = \{F \subseteq E : |F \cap E_i| \leq b_i, i = 1, \dots, k\})$ is a matroid (the so called partition matroid). Especially the matroids in exercise 8.4 are partition matroids.

2. A (directed) path in a digraph $D = (V, A)$ is called hamilton path, if it meets all vertices of D. Reformulate the problem of finding a hamilton path in the fashion of a search for a maximum cardinality independent set in the intersection of three matroids.

3. Let $T = \{t_1, \dots, t_n\}$ be a set and $A_i \in 2^T$, $i = 1, \dots, m$ ($A_i = A_j$ for different i, j allowed). If there is an injective map $\varphi : I \to \{1, \dots, n\}$ with $t_{\varphi(i)} \in A_i$ (choice function) exists for a subfamily $\mathcal{A}_I = \{A_i : i \in I\}$ with $I \subseteq \{1, \dots, m\}$ then $T_I = \{t_{\varphi(i)} : i \in I\}$ is called *transversal* or *system of distinct representatives* (SDR) of $\mathcal{A}_I$.

   Prove $(T, \mathcal{F} = \{T_I : \exists I \subseteq \{1, \dots, m\} : T_I \text{ is transversal of } \mathcal{A}_I\})$ is a matroid (the so called *transversal matroid*).

   Hint: Construct a bipartite graph $(V = \mathcal{A}_{\{1,\dots,m\}} \cup T, E = \{\{A_i, t_j\} : t_j \in A_i\})$; transversals are represented as matchings; if $X, Y \in \mathcal{F}$ with $|X| < |Y|$, then the union of the representing matchings contains an alternating path which enables to enlarge X.

4. In an enterprise n persons apply for m jobs. Every person is qualified for a subset of all jobs only, but has a value for the enterprise independently of the job.

   Create a greedy algorithm on an independence system to search for an employment of maximum value. Which qualtity warranty is possible?

5. Present the systems of cycles and cocycles of the following graph: