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Introduction to Discrete Mathematics
Exercise 8

1. A coloring of a graph $G = (V, E)$ is a map $f : V \to C$ (color set), such that $uv \in E$ implies $f(u) \neq f(v)$. The chromatic number $\chi(G)$ is the smallest number of colors needed for a coloring of G. Determine $\chi(K_n)$, $\chi(K_{m,n})$, $\chi(Q_n)$, $\chi(P_n)$ (path), $\chi(C_n)$ (Cycle).

2. Determine a minimal spanning tree for the following list of edges and edge weights of a graph on 14 vertices:

1 2 4.	2 3 5.	4 5 2.	6 7 2.	8 13 3.	10 11 5.
1 8 6.	2 14 3.	5 6 7.	6 13 6.	9 10 1.	10 12 3.
1 11 7.	3 4 1.	5 13 6.	7 8 6.	9 11 5.	11 12 6.
1 13 1.	3 14 1.	5 14 5.	7 9 3.	9 12 6.	13 14 2.

3. Let $G = (V, E)$ be an undirected graph with edge weights $w_e \in \mathbb{R}_+$ for $e \in E$. A *hamiltonian cycle* of G is a cycle covering V. The *Travelling Salesman Problem*, TSP is to determine a hamiltonian cycle of minimum weight. Give a formulation of this problem as a minimization problem over an independence system.

4. Prove the following:
a) Let $G = (V, E)$ be an undirected graph, $S \subseteq V$ be an independent set of G, and $k_s \in \mathbb{N}_0$ for all $s \in S$. Let $\delta_F(s)$ denote the set $\{e \in F : s \in e\}$. Then $(E, \mathcal{F} = \{F \subseteq E : |\delta_F(s)| \leq k_s \ \forall s \in S\})$is a matroid.
b) Let $D = (V, A)$ be a directed graph, $S \subseteq V$, $k_s \in \mathbb{N}_0$ for all $s \in S$. Let $\delta_F^-(s)$ denote the set $\{a \in A : a = (u, s)\text{for a}u \in V\}$. Then $(A, \mathcal{F} = \{F \subseteq A : |\delta_F^-(s)| \leq k_s \ \forall s \in S\})$ is a matroid.

5. Describe the systems of circuits, bases and the independence system of the matrix matroid with respect to to the columns of the following matrix:

$$A = \begin{pmatrix} 1 & -1 & 2 & 1 & -1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 & -1 \end{pmatrix}$$