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Chemnitz, november 17th 2003
due: november 24th 2003

Introduction to Discrete Mathematics
Exercise 6

1. Prove, that any two longest paths of a connected graph share a common vertex!

2. Let $G = (V, E)$. For given $V' \subseteq V$ the set of vertices in $V \setminus V'$ having at least one neighbor in V' is denoted by $R(V')$. Prove $b(G) \geq \max\limits_{1 \leq s \leq |V|} \min\limits_{|V'|=s} |R(V')|$!

3. Prove that an edge is a bridge if and only if it is not contained in a cycle!
   In which graphs the edge set contains bridges only?
   Prove that a graph is bridgeless, if it contains vertices of even degree only.

4. The complete bipartite graph $K_{1,n-1}$ is called *star*.
   Prove or disprove:
   a) If G has diameter 2 then G contains a spanning star (a star covering $V(G)$).
   b) If G contains a spanning star, then the diameter of G is two.

5. For a graph $G = (V, E)$ the *complementary graph* (denoted by $\overline{G}$) is the graph $(V, E = \binom{V}{2} \setminus E)$. Prove for a k-regular graph G on n vertices:
   The numbers of triangles in G and $\overline{G}$ add up to $\binom{n}{3} - \frac{n}{2}k(n-k-1)$, exactly.