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Introduction to Discrete Mathematics
Exercise 5

1. Prove the following equations:
   (a) $O(f(n))O(g(n)) = O(f(n)g(n))$,
   (b) Falls $f(n) > 0\ \forall n \in \mathbb{N}$, dann $O(f(n)g(n)) = f(n)O(g(n))$,
   (c) $O(f(n)) + O(g(n)) = O(|f(n)| + |g(n)|)$.

2. Let $f(n) = n^2$ (n even) and $f(n) = 2n$ (n odd). Verify
   - $f(n) = O(n^2)$,
   - not $f(n) = o(n^2)$, and
   - not $n^2 = O(f(n))$.

3. Let $T(n) = 2T(\lfloor\sqrt{n}\rfloor) + \lg n$. Verify (for suitable powers of 2) $T(n) = O(\lg n \lg\lg n)$.

4. Suppose, there's an n-step-algorithm for input length n. Suppose further, step i uses i^2 operations. Verify, that the running time of the algorithm is $O(n^3)$.

5. Usually, multiplication of $n \times n$ matrices needs $\Theta(n^3)$ flops (flowting point operations '$*$' and '$+$'), especially 8 multiplications for $n = 2$. The following method of **Strassen** works with only 7 multiplications:
   Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $B = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$. Prove, that the elements of AB are sums of terms $\pm m_i$ sind, while $m_1 = (a+d)(\alpha+\delta)$, $m_2 = (c+d)\alpha$, $m_3 = a(\beta-\delta)$, $m_4 = d(\gamma-\alpha)$, $m_5 = (a+b)\delta$, $m_6 = (a-c)(\alpha+\beta)$, $m_7 = (b-d)(\gamma+\delta)$. How many additions/substractions are contained in the usual calculation? How additions/substractions uses Strassens method? Find a method, to calculate the product of two $n \times n$-matrices using only $\Theta(n^{\log_2 7})$ flops.
   Hint: Quarter the matrices and use recursion starting at $n = 2$.