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Introduction to Discrete Mathematics
Exercise 4

1. How many positive integers ≤ 1000000 are of the form n^k where n and k are integers and $k > 2$?

   Hints: Why can we think about k as a prime? How many such integers > 1 do we find?

2. Generalize the principle of inclusion and exclusion:
   Let $P_1, \ldots, P_m$ be properties of elements of an n element set S. Prove that the number of elements satisfying exactly t of this properties equals

$$\sum_{i_1<\ldots<i_t} N(P_{i_1} \ldots P_{i_t}) - \binom{t+1}{t} \sum_{i_1<\ldots<i_{t+1}} N(P_{i_1} \ldots P_{i_{t+1}}) + \ldots \pm \binom{m}{t} N(P_1 \ldots P_m)$$

3. Calculate $\sum_{k=1}^{n}(-1)^k k$ and $\sum_{k=1}^{n}(-1)^k k^2$!

   Hint: Use *isolation of terms*!

4. Prove that for every positive integer n there is a unique finite sequence $(m_1, \ldots, m_t)$ of integers such that $n = F_{m_1} + \ldots F_{m_t}$, $m_i \geq m_{i+1} + 2$, and $m_t \geq 2$ holds.

5. Let A_n be the number of possibilities to fill a $2 \times n$-rectangle with nonintersecting 1×2 dominoes. Find a recursion for A_n and calculate A_n explicitly!

   Hint: It is possible to turn the dominoes!