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Introduction to Discrete Mathematics
Exercise 14

1. Show that the running time of Heapsort is $O(n \log n)$.

2. Sort the sequence 0,8,15,3,7,42,47,11 by Mergesort, Quicksort, and Heapsort and count the number of actually used comparisons.

3. Write a program for a deterministic touring machine, which adds 1 to a given non-negative integer (given as binary number wiht most significant bit at position 1) while it does not accept an empty input.

   Test the program for the numbers five and three.

4. The knapsack problem is to choose a subset S of a set of n given objects $1, \ldots, n$ with weights $w_1, \ldots w_n$ and values $c_1, \ldots, c_n$ such that the value $\sum_{i \in S} c_i$ of the whole set S is maximal while the weight $\sum_{i \in S} w_i$ of the whole set S does not exceed the capacity k of the knapsack.

   Show: An algorithm solving the knapsack problem can easy be used to solve PARTITION.

5. Prove the equivalency of the following statements for a given graph $G = (V, E)$ and a subset V' of its vertex set:

   (i) V' is a vertex cover of G (i.e., $\{u, v\} \in E \Rightarrow u \in V'$ oder $v \in V'$).
   (ii) $V - V'$ is an independent set of G.
   (iii) $V - V'$ induces a clique (complete subgraph) in the complementary graph $\overline{G}$.

   Conclude from it, that a polynomial algorithm (with respect to $|V|$ and $|E|$) for finding a minimal vertex cover also gives a polynomial algortihm for finding a maximal clique, and vice versa.