## Introduction to Discrete Mathematics <br> Exercise 11

1. Run Floyd's algorithm for the following directed graphs! How can one construct the shortest paths from the informations produced by this run?

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 6 | 5 |  |  |
| 2 |  |  | 7 | 3 | -2 |
| 3 |  |  |  | -4 | 8 |
| 4 |  | -1 |  |  |  |
| 5 | 2 |  |  | 7 |  |

2. Calculate a maximum flow from $s$ to $t$ in the following network. Use the algorithm of Ford and Fulkerson. Always choose the shortest augmenting paths. Which edges are contained in the minimal cut?

3. Solve the previous example by the algorithm of Fujishige. How to find a minimal cut here?
4. Find a maximal matching for the following bipartite graph by constructing a maximal flow.

5. Let $G=(V, E)$ be an undirected graph and $s$ and $t$ be two of its vertices. Construct an algorithm finding the maximum number of internally vertexdisjoint paths from $s$ to $t$ !
Hint: Replace $G$ by a directed graph $D$ which contains two vertices $v_{i n}$ and $v_{\text {out }}$ for each vertex $v$ of $G$, which are connected by an arc.
