Rainbow connection and size of graphs

Ingo Schiermeyer
Technische Universität Bergakademie Freiberg

An edge-coloured connected graph G is called *rainbow-connected* if each pair of distinct vertices of G is connected by a path whose edges have distinct colours. The *rainbow connection number* of G, denoted by $rc(G)$, is the minimum number of colours such that G is rainbow-connected. In this talk we consider the following problem:

Problem For all integers n and k with $1 \leq k \leq n-1$ compute and minimize the function $f(n,k)$ with the following property: If $|V(G)| = n$ and $|E(G)| \geq f(n,k)$ then $rc(G) \leq k$.

For n and k with $1 \leq k \leq n-1$ it holds that $f(n,k) \geq \binom{n-k+1}{2} + k - 1$. It has been shown that $f(n,k) = \binom{n-k+1}{2} + k - 1$ for $k = 1, 2, 3, 4$ and for $n - 6 \leq k \leq n - 1$.

In this talk we will report about these results and show some further recent progress obtained for this problem.