

## Solution 4: Rate coded neurons

### A. Theoretical evaluation of a winner-takes-all network

**Question 1** (points: 2): *For which  $p, q$  values do we see a response of some of the cells  $n$  due to an input?*

**Answer 1:** Let us first consider the steady state of a neuron  $i$ :

$$z_i + q \cdot \sum_{j=1}^n [f(z_j)]^+ = I_i^{ext} + p [f(z_i)]^+ \quad (1)$$

Since we want a response even for small inputs, we estimate  $I_i^{ext} = 0$ . We know that  $z_i < f(z_i)$ , thus in the limit we obtain:

$$q < p \quad (2)$$

**Question 2** (points: 3): *For which  $p, q$  values does this network guarantee that only a single cell is active (winner-takes-all behavior)?*

**Answer 2:** To ensure that only a single neuron fires, there has to be a winning neuron  $w$  with the rate  $z_w$  that inhibits all other neurons such that they will not fire anymore. Thus, we estimate that the inhibitory term is only driven by the winning neuron  $w$  and the neuron  $i$ , since more active neurons would make the inhibition even larger. With these assumptions the ODE of a neuron  $i$  writes:

$$\tau \frac{\delta z_i}{\delta t} = -z_i + I_i^{ext} + (p - q) [f(z_i)]^+ - q \cdot [f(z_w)]^+ \quad (3)$$

As long as the change in the firing rate of the neuron  $i$  is negative, sooner or later this neuron will be inactive:

$$0 > -z_i + I_i^{ext} + (p - q) [f(z_i)]^+ - q \cdot [f(z_w)]^+ \quad (4)$$

We can further simplify the expression by setting  $z_i = 0$  and  $I_i^{ext} = 1$ :

$$0 > 1 + (p - q) [f(z_i)]^+ - q \cdot [f(z_w)]^+ \quad (5)$$

Since  $(p-q) > 0$  as required to have activity in the network at all, we can estimate  $[f(z_i)]^+$  with the largest possible value which is  $[f(z_w)]^+$  which leads us to the answer:

$$q > \frac{1}{2} \left( p + \frac{1}{[f(z_w)]^+} \right) \quad (6)$$

When we assume that  $c$  is small (e.g.  $c = 0.1$ ) we can furthermore estimate  $[f(z_w)]^+$  with 1 which leads to the final answer:

$$q > \frac{1}{2}(p + 1) \quad (7)$$

## B. Emperical evaluation of a winner-takes-all network

**Answer 2:** The last step of the solution to A.2 was to estimate  $[f(z_w)]^+$  with 1. Depending on the choice of  $c$ ,  $[f(z_w)]^+$  might be much smaller than 1. When for example  $[f(z_w)]^+ < 0.5$  it follows from 6:

$$q > \frac{p}{2} + 1 \quad (8)$$

Previously we estimated

$$q > \frac{1}{2}(p + 1) \quad (9)$$

If  $q$  is chosen such that it obeys both constraints and we estimate  $z_i = 0$ ,  $I_i^{ext} = 1$  and  $z_i = z_w$ , the ODE becomes

$$1 + (p - q) [f(z_w)]^+ - q \cdot [f(z_w)]^+ = \quad (10)$$

$$1 + p [f(z_w)]^+ - 2q [f(z_w)]^+ < \quad (11)$$

$$1 + p [f(z_w)]^+ - p [f(z_w)]^+ - 2 [f(z_w)]^+ = 1 - 2 [f(z_w)]^+ \quad (12)$$

If  $[f(z_w)]^+ < 0.5$  this is positive and thus  $z_i$  is raising.