# Exercise 3: Rate coded neurons

## A. Theoretical evaluation of a winner-takes-all network

Consider a network of neurons $i \in \{1, ..., n\}$ that is described by the following equation:

$$\tau \frac{\delta z_i}{\delta t} = -z_i + I_i^{ext} + p\,[f(z_i)]^+ - q * \sum_{j=1}^{n} [f(z_i)]^+ \quad (1)$$

$$f(z) = \frac{e^{\frac{z}{c}} - 1}{e^{\frac{z}{c}} + 1}$$

where $z >= 0$ represents the firing rate of a cell, $[]^+$ the positive value of the argument and $p, q$ weights of connections between neurons. Assume that the external inputs $I_i^{ext}$ of the cells $i$ are between zero and one.

**Question 1** (points: 2)**:** *For which p, q values do we see a response of some of the cells n due to an input?*

Hint: Consider the steady state of a neuron $i$ and ensure that the condition is fulfilled regardless of the exact input.

**Question 2** (points: 3)**:** *For which p, q values does this network guarantee that only a single cell is active (winner-takes-all behavior)?*

Hint: Write down the ODE of a neuron $i$ and make estimates. How many neurons do you want to consider that inhibit the cell $i$? What should be the change in firing rate of the cell $i$?

## B. Empirical evaluation of a winner-takes-all network

(5 points)

Test your answers to the questions from part A empirically by implementing a neural model using equation 1. The activity of all cells should be positive. Set the time constant to $\tau = 10ms$. Try to set $c$ to a good value by plotting $f(z)$ for different values of $c$. Give a short reason for your choice of $c$.

**Question 1** (points: 3)**:** *Set $p$ and $q$ according to your solution from part A (Question 1 and 2). Activate your model with the input $I_i^{ext}$ and observe the response of the neurons. Do the cells respond as you have predicted from your theoretical analysis? Show that the network does not respond in the same way if the constraint you have found in part A is violated.*

**Question 2** (points: 2)**:** *Set $p$ and $q$ again according to your solution from part A. Activate your model with an input $I_i^{ext}$. Test the influence of the parameter $c$ in $f(z)$. Let us consider only a network with two neurons. Try to find a network input and parameters $p$, $q$ which fulfill your constraint from part A such that the network does* **not** *show winner-takes-all behavior. Is this possible?*

Hint: Let us define that the cell with the larger input is cell $w$ and the other cell is cell $i$. Set $c = 1$, and $q$ such that $1 + \frac{p}{2} > q > \frac{1}{2}(1 + p)$ and $z_i(t = 0) \leq z_w(t = 0)$ close to zero and $I_i^{ext}$ close to one.