# Model neurons

## Integrate-and-Fire models

*Suggested reading:*

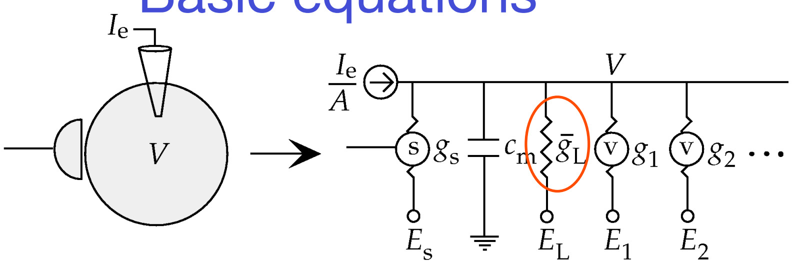
Chapter 5.4 in Dayan, P. & Abbott, L., Theoretical Neuroscience, MIT Press, 2001.

## *Model neurons: Integrate- and fire models*

*Contents:*

- Generation of spikes
- Interspike-intervall firing rate
- Spike-rate adaptation
- Refractory period

# Basic equations




Basic equation for a single compartment model:

$$c_m \frac{dV(t)}{dt} = -i_m + \frac{I_e(t)}{A}$$

Model a single passive leakage term:

$$i_m = \bar{g}_L (V(t) - E_L)$$

Membrane potential:

$$c_m \frac{dV(t)}{dt} = -\bar{g}_L (V(t) - E_L) + \frac{I_e(t)}{A}$$

Basic equation for the passive integrate-and-fire model:

$$\tau_m \frac{dV(t)}{dt} = E_L - V(t) + R_m I_e(t)$$

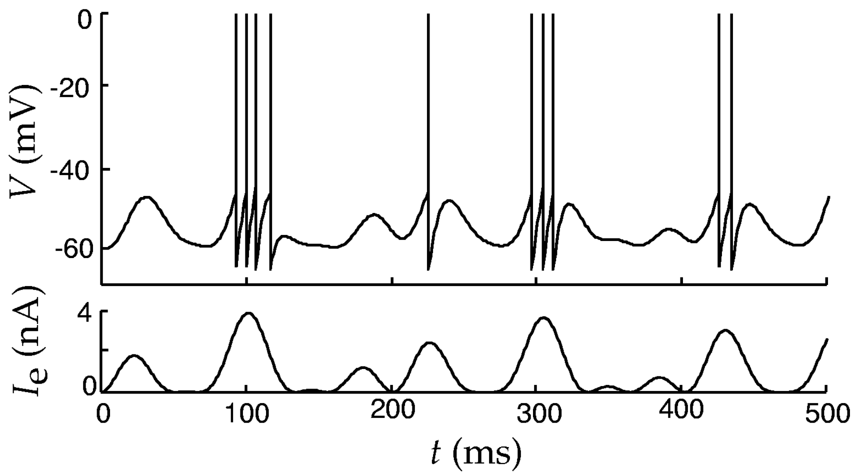


# Generation of spikes

Fire a spike if $V_m(t) = V_{th}$

and reset the membrane potential $V_{reset} < V_{th}$

Example: Passive inegrate-and-fire model driven by a time-varying current



# Interspike-interval firing rate

Interspike-interval firing rate: $r_{isi}$

Suppose a neuron has fired at t=0, then $V(0) = V_{reset}$

The next spike will then occur at time $t_{isi}$ when the membrane potential reaches

$$V(t_{isi}) = V_{th}$$

Response to a constant injected current:

$$V(t) = v_0 e^{\frac{-t}{\tau_m}} + v_1$$

$$V(t) = E_L + R_m I_e + (V(t=0) - E_L - R_m I_e) e^{\frac{-t}{\tau_m}}$$

$$V(t_{isi}) = E_L + R_m I_e + (V_{reset} - E_L - R_m I_e) e^{\frac{-t_{isi}}{\tau_m}}$$



# Interspike-interval firing rate

Interspike-interval firing rate: $r_{isi}$

Suppose a neuron has fired at t=0, then $V(0) = V_{reset}$

The next spike will then occur at time $t_{isi}$ when the membrane potential reaches

$$V(t_{isi}) = V_{th}$$

$$V(t_{isi}) = E_L + R_m I_e + (V_{reset} - E_L - R_m I_e) e^{\frac{-t_{isi}}{\tau_m}}$$

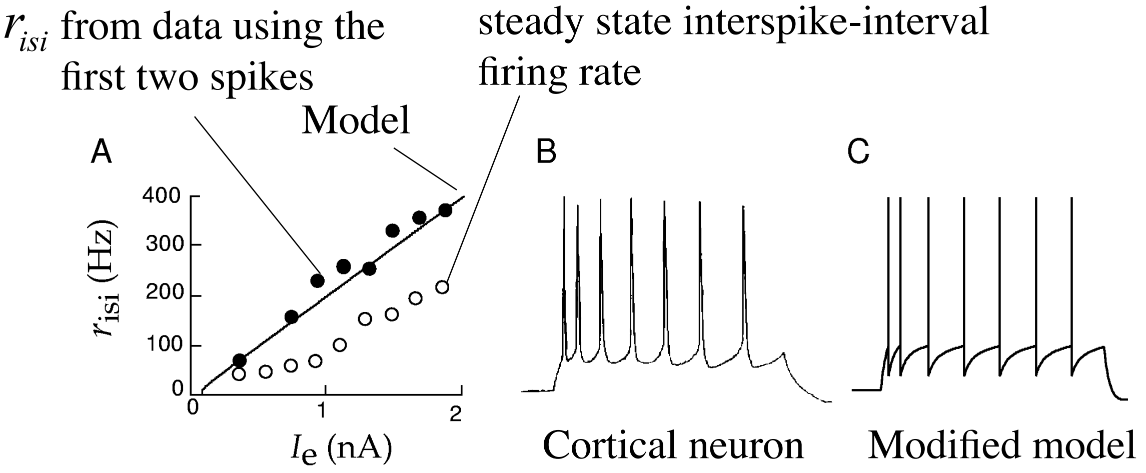
$$r_{isi} = \frac{1}{t_{isi}} = \left( \tau_m \ln\left( \frac{R_m I_e + E_L - V_{reset}}{R_m I_e + E_L - V_{th}} \right) \right)^{-1}$$

for

$$R_m I_e > V_{th} - E_L$$

# Interspike-interval firing rate

Example:




The first two spikes indicate a larger interspike-interval firing rate than the one that occurs in the steady state.



# Spike-rate adaptation

Include an additional current in the model

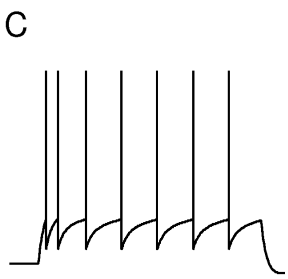
$$\tau_m \frac{dV}{dt} = E_L - V - r_m g_{sra}(V - E_K) + R_m I_e(t)$$

The conductance $g_{sra}$ relaxes exponentially to 0 if no spike occurs

$$\tau_{sra} \frac{dg_{sra}}{dt} = -g_{sra}$$

Whenever the neuron fires a spike $g_{sra}$ is increased:

$$g_{sra} \rightarrow g_{sra} + \Delta g_{sra}$$




Model with spike-rate adaptation

# Refractory period

3 methods:

- Absolute refractory period ($T_{refract}$)
- Add an additional current (conductance)
- Raise the threshold after an action potential, and allow to relax it to its normal value



# Dynamic refractory period

$$\tau_m \frac{dV}{dt} = E_L - V - r_m g_{ref} (V - E_{ref}) + R_m I_e(t)$$

The conductance $g_{ref}$ relaxes exponentially to 0

$$\tau_{ref} \frac{dg_{ref}}{dt} = -g_{ref}$$

Whenever the neuron fires a spike $g_{ref}$ is increased:

$$g_{ref} \rightarrow g_{ref} + \Delta g_{ref}$$

But: $\Delta g_{ref} > \Delta g_{sra}$

$\tau_{ref} < \tau_{sra}$

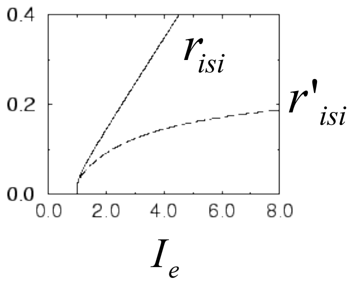
==> clamp V to $E_{ref}$

# Absolute refractory period

Interspike interval without refractory period:

Simplification $E_L = V_{reset} = 0$

$$r_{isi} = \frac{1}{t_{isi}} = \left( \tau_m \ln\left( \frac{R_m I_e}{R_m I_e - V_{th}} \right) \right)^{-1}$$




Interspike interval with absolute refractory period:

$$r'_{isi} = \frac{1}{t_{isi} + T_{refract}} = \left( T_{refract} + \tau_m \ln\left( \frac{R_m I_e}{R_m I_e - V_{th}} \right) \right)^{-1}$$