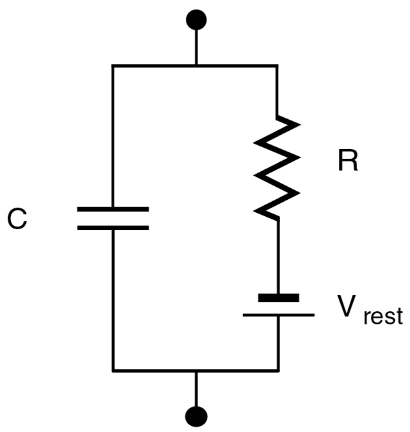
Model neurons

Electrical circuits

Suggested reading:

Chapter A.4 in Dayan, P. & Abbott, L., Theoretical Neuroscience, MIT Press, 2001.

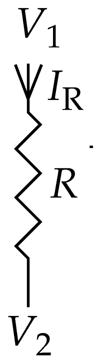
Model neurons: Electrical circuits



Contents:

- Ohm's law
- Capacitor
- Kirchhoff's law

Ohm's law



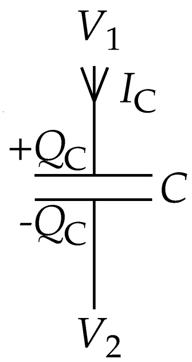
Current I_R flows through a resistance R producing a voltage drop $V_1 - V_2 = V_R$.

Biophysical models of single cells involve equivalent circuits composed of resistors, capacitors, and voltage and current sources.

A resistor satisfies Ohm's law, which states that the voltage $V_R = V_1 - V_2$ across a resistance R carrying a current I_R is $V_R = I_R R$.

Resistance is measured in ohms (Ω) defined as the resistance through which one ampere of current causes a voltage drop of one volt (1 V = 1 A × 1 Ω).

Capacitor



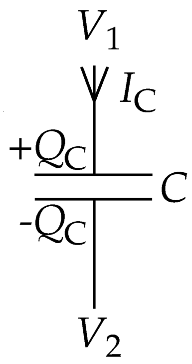
Charge $\pm Q_C$ is stored across a capacitance C leading to a voltage $V_C = V_1 - V_2$ and a current I_C.

A capacitor stores charge across an insulating medium, and the voltage across it $V_C = V_1 - V_2$ is related to the charge it stores Q_C by $CV_C = Q_C$ where C is the capacitance.

Electrical current cannot cross the insulating medium, but charges can be redistributed on each side of the capacitor, which leads to the flow of current.

We can take a time derivative of both sides of the eq. above and use the fact that current is equal to the rate of change of charge, $I_C = d\,Q_C/dt$, to obtain the basic voltage-current relationship for a capacitor,

Capacitor



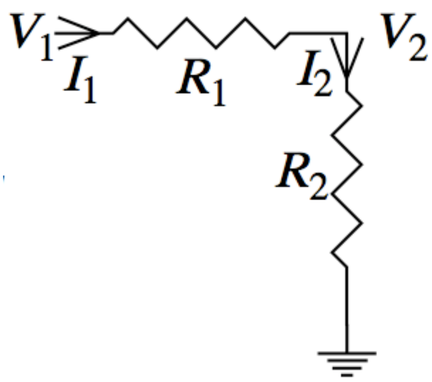
Charge $\pm Q_C$ is stored across a capacitance C leading to a voltage $V_C = V_1 - V_2$ and a current I_C.

Basic voltage-current relationship for a capacitor:

$$C\frac{dV_C}{dt} = I_C$$

Capacitance is measured in units of farads (F) defined as the capacitance for which one ampere of current causes a voltage change of one volt per second (1 F × 1 V/s = 1 A).

Kirchhoff's law



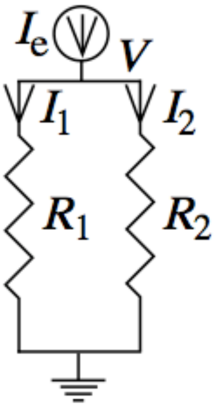
Series resistor circuit called a voltage divider.

Kirchoff's laws state that:

- voltage differences around any closed loop in a circuit must sum to zero
- the sum of all the currents entering any point in a circuit must be zero

Applying the second of these rules to the circuit on the left, we find that $I_1 = I_2$. Ohm's law tells us that $V_1 - V_2 = I_1 R_1$ and $V_2 = I_2 R_2$. Solving these gives $V_1 = I_1(R_1 + R_2)$, which tells us that resistors arranged in series add, and $V_2 = V_1 R_2 / (R_1 + R_2)$, which is why this circuit is called a voltage divider.

Kirchhoff’s law



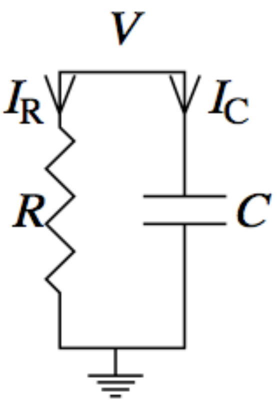
Parallel resistor circuit. I_e represents an external current source.

For the circuit on the left, Kirchoff ’s and Ohm’s laws tells us that

$I_e = I_1 + I_2 = V / R_1 + V / R_2$.

This indicates how resistors add in parallel, $V=I_e R_1 R_2 / (R_1 + R_2)$.

Kirchhoff’s law



RC circuits: Current I_C=-I_R flows in the resistor-capacitor circuit as the stored charge is released.

Kirchoff’s laws require that $I_C + I_R = 0$:

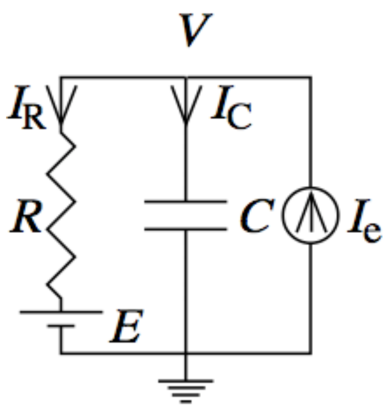
$$C\frac{dV}{dt} = I_C = -I_R = -\frac{V}{R}$$

Solving this, gives

$$V(t) = V(0)\exp(-t/RC)$$

showing the exponential decay (with time constant $\tau = RC$) of the initial voltage V(0) as the charge on the capacitor leaks out through the resistor.

Kirchhoff's law



RC circuits: = Simple passive membrane model including a potential E and current source I_e.

Two extra components needed to build a simple model neuron, the voltage source E and the current source I_e . Using Kirchoff 's laws, $I_e - I_C - I_R = 0$, and the equation for the voltage V is

$$C\frac{dV}{dt} = \frac{E - V}{R} + I_e$$

If I_e is constant, the solution of this equation is

$$V(t) = V_\infty + (V(0) - V_\infty)\exp(-t/\tau)$$

where $V_\infty = E + R\, I_e$ and τ = RC. This shows exponential relaxation from the initial potential V(0) to the equilibrium potential V_∞ at a rate governed by the time constant τ of the circuit.