Übung 9

Beispiel zu HMM

02.02.12

HMM 1

für (b,e,e,r,e)

3 Zustände

3 Beobachtungen b, e, r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

HMM 1

(b,e,e,r,e)

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

(b,e,e,r,e)

$$\mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

kein b

HMM 2

für (r,e,b,e)

3 Zustände

3 Beobachtungen b, e, r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

HMM 2

(r,e,b,e)

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

(r,e,b,e)

$$\mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

kein r

Beobachtungen

$$o^1 = (r, e, b, e)$$

$$o^2 = (b, e, e, r, e)$$

$$o^3 = (r, e, e, b, e)$$

$$o^4 = (b, e, r, e)$$

Auftrittswahr. für Beobachtung 1

$$o^1 = (r, e, b, e)$$

$$p(o^1 \mid \lambda_1)$$ → (b,e,e,r,e)

$$p(o^1 \mid \lambda_2)$$ → (r,e,b,e)

In beiden Fällen wollen wir zum Zeitpunkt 4 im Endzustand 3 sein.

Wir erwarten: $$p(o^1 \mid \lambda_2) > p(o^1 \mid \lambda_1)$$

Summierung über alle möglichen Zustandsfolgen – HMM1

$$o^1 = (r, e, b, e)$$

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

$$P(o^1 \mid \lambda_1) = \sum_{s \in S^T} e_{s_1} b_{s_1 o_1} \prod_{n=2}^{T} a_{s_{n-1} s_n} b_{s_n o_n}$$

Mögliche Zustandsfolgen: $s^1 = (1,1,2,3)$ $s^2 = (1,2,2,3)$

nicht: $(1,1,1,1)$ $(1,1,1,2)$ $(1,1,2,2)$

$(1,2,3,3)$ ist nicht möglich (Zustand 3 kann kein b ausgeben)

Summierung über alle möglichen Zustandsfolgen – HMM1

$$o^1 = (r, e, b, e)$$

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

$$P(o^1 \mid \lambda_1) = \sum_{s^1, s^2} e_{s_1} b_{s_1 o_1} \prod_{n=2}^{4} a_{s_{n-1} s_n} b_{s_n o_n}$$

$$s^1 = (1,1,2,3)$$

$$s^2 = (1,2,2,3)$$

$$= [1 \cdot 0.1][0.1 \cdot 0.1][0.9 \cdot 0.1][0.8 \cdot 0.6] + [1 \cdot 0.1][0.9 \cdot 0.8][0.2 \cdot 0.1][0.8 \cdot 0.6]$$

$$= 4.32 \cdot 10^{-5} + 6.91 \cdot 10^{-4} = 7.34 \cdot 10^{-4}$$

$$P(o^1 \mid \lambda_1) = 7.34 \cdot 10^{-4}$$

Berechnung mit Vorwärtsalgorithmus

$$o^1 = (r, e, b, e)$$

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

$$\alpha_1(1) = 1 \cdot 0.1 = 0.1 \qquad \alpha_1(2) = \alpha_1(3) = 0$$

$$\alpha_{n+1}(i) = b_{io_{n+1}} \cdot \sum_{j=1}^{3} \alpha_n(j) \cdot a_{ji}$$

$$\alpha_2(1) = 0.1 \cdot (0.1 \cdot 0.1) = 0.001$$

$$\alpha_2(2) = 0.8 \cdot (0.1 \cdot 0.9) = 0.072$$

$$\alpha_2(3) = 0.6 \cdot (0.1 \cdot 0) = 0$$

$$\alpha_3(1) = 0.8 \cdot (0.001 \cdot 0.1 + 0.072 \cdot 0) = 8 \cdot 10^{-5}$$

$$\alpha_3(2) = 0.1 \cdot (0.001 \cdot 0.9 + 0.072 \cdot 0.2) = 0.00153$$

$$\alpha_3(3) = 0 \cdot (\cdots) = 0$$

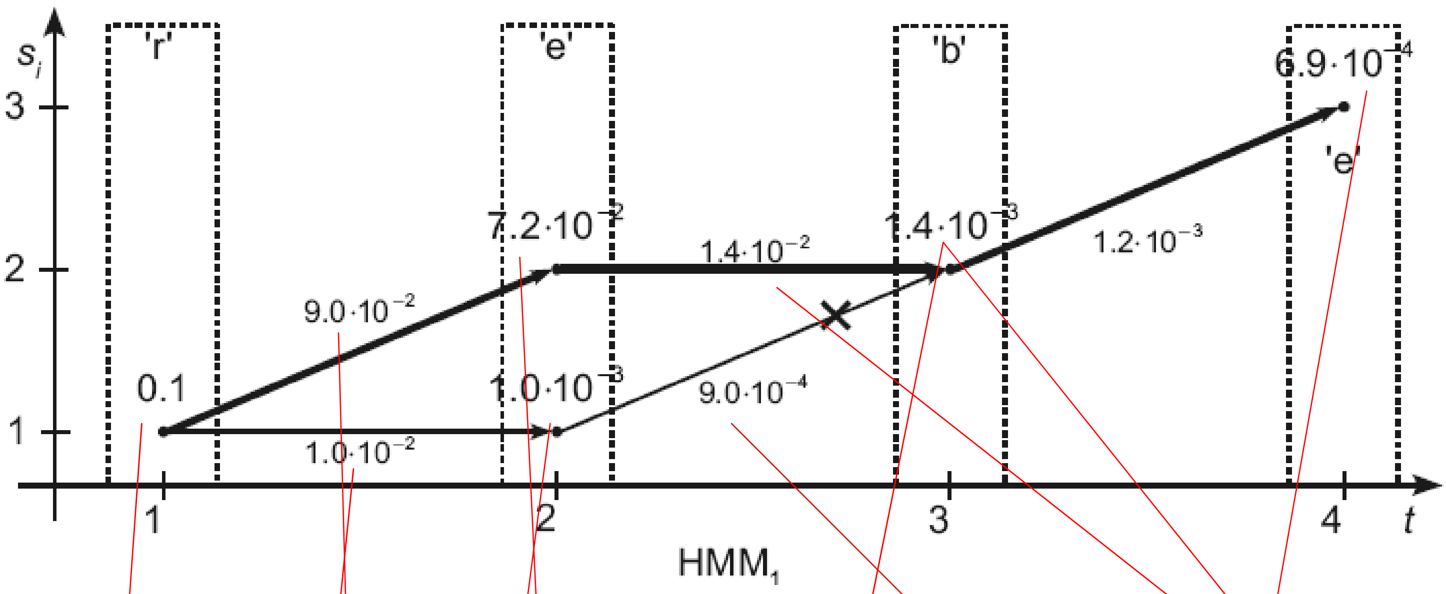
$$\alpha_4(3) = 0.6 \cdot (\alpha_3(1) \cdot 0 + \alpha_3(2) \cdot 0.8 + \alpha_3(3) \cdot 1) = 7.34 \cdot 10^{-4}$$

Berechnung mit Vorwärtsalgorithmus

$$\alpha_4(1) = 0.1 \cdot (\alpha_3(1) \cdot 0.1 + \alpha_3(2) \cdot 0 + \alpha_3(3) \cdot 0) = 8 \cdot 10^{-7}$$

$$\alpha_4(2) = 0.8 \cdot (\alpha_3(1) \cdot 0.9 + \alpha_3(2) \cdot 0.2 + \alpha_3(3) \cdot 0.8) = \ldots > 0$$

Optimale Folge – Viterbi Alg.



$$\alpha_1(1) = 1 \cdot 0.1 = 0.1$$

$$\alpha_2(1) = 0.1 \cdot (0.1 \cdot 0.1) = 0.001$$

$$\alpha_2(2) = 0.8 \cdot (0.1 \cdot 0.9) = 0.072$$

$$\alpha_3(2) = 0.1 \cdot (0.001 \cdot 0.9 + 0.072 \cdot 0.2) = 0.00153$$

$$p(o^1 \mid \lambda_1) = 4.32 \cdot 10^{-5} + 6.91 \cdot 10^{-4} = 7.34 \cdot 10^{-4}$$

Optimale Folge

$$o^1 = (r, e, b, e)$$

(b,e,e,r,e)

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

Optimale Folge: 1 2 2 3

Summierung über alle möglichen Zustandsfolgen – HMM2

$$o^1 = (r, e, b, e)$$

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

$$P(o^1 \mid \lambda_2) = \sum_{s \in S^T} e_{s_1} b_{s_1 o_1} \prod_{n=2}^{T} a_{s_{n-1} s_n} b_{s_n o_n}$$

Mögliche Zustandsfolgen: $s^1 = (1,1,1,3)$ $s^2 = (1,1,2,3)$ $s^3 = (1,2,2,3)$

$s^4 = (2,2,2,3)$ $s^5 = (1,1,3,3)$ $s^6 = (1,2,3,3)$ $s^7 = (2,2,3,3)$

$s^8 = (1,3,3,3)$ $s^9 = (2,3,3,3)$

Summierung über alle möglichen Zustandsfolgen – HMM2

$$p_2(\mathbf{o}_1,\mathbf{q}_1|\lambda_2)=\underbrace{0.5\cdot0.8\cdot0.1\cdot0.1\cdot0.1\cdot0.1\cdot0.1\cdot0.5}_{\mathbf{q}_1=(s_1,s_1,s_1,s_3)}=2\cdot10^{-6}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_2|\lambda_2)=\underbrace{0.5\cdot0.8\cdot0.1\cdot0.1\cdot0.8\cdot0.1\cdot0.9\cdot0.5}_{\mathbf{q}_2=(s_1,s_1,s_2,s_3)}=1.44\cdot10^{-4}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_3|\lambda_2)=\underbrace{0.5\cdot0.8\cdot0.8\cdot0.8\cdot0.1\cdot0.1\cdot0.9\cdot0.5}_{\mathbf{q}_3=(s_1,s_2,s_2,s_3)}=1.15\cdot10^{-3}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_4|\lambda_2)=\underbrace{0.5\cdot0.1\cdot0.1\cdot0.8\cdot0.1\cdot0.1\cdot0.9\cdot0.5}_{\mathbf{q}_4=(s_2,s_2,s_2,s_3)}=1.8\cdot10^{-5}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_5|\lambda_2)=\underbrace{0.5\cdot0.8\cdot0.1\cdot0.1\cdot0.1\cdot0.5\cdot1\cdot0.5}_{\mathbf{q}_5=(s_1,s_1,s_3,s_3)}=1\cdot10^{-4}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_6|\lambda_2)=\underbrace{0.5\cdot0.8\cdot0.8\cdot0.8\cdot0.9\cdot0.5\cdot1\cdot0.5}_{\mathbf{q}_6=(s_1,s_2,s_3,s_3)}=5.76\cdot10^{-2}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_7|\lambda_2)=\underbrace{0.5\cdot0.1\cdot0.1\cdot0.8\cdot0.9\cdot0.5\cdot1\cdot0.5}_{\mathbf{q}_7=(s_2,s_2,s_3,s_3)}=9\cdot10^{-4}$$

$$P(o^1\mid\lambda_2)=7.05\cdot10^{-2}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_8|\lambda_2)=\underbrace{0.5\cdot0.8\cdot0.1\cdot0.5\cdot1\cdot0.5\cdot1\cdot0.5}_{\mathbf{q}_8=(s_1,s_3,s_3,s_3)}=5\cdot10^{-3}$$

$$p_2(\mathbf{o}_1,\mathbf{q}_9|\lambda_2)=\underbrace{0.5\cdot0.1\cdot0.9\cdot0.5\cdot1\cdot0.5\cdot1\cdot0.5}_{\mathbf{q}_9=(s_2,s_3,s_3,s_3)}=5.63\cdot10^{-3}$$

Berechnung mit Vorwärtsalgorithmus

$$o^1 = (r, e, b, e)$$

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

$$\alpha_1(1) = 0.5 \cdot 0.8 = 0.4$$

$$\alpha_1(2) = 0.5 \cdot 0.1 = 5 \cdot 10^{-2}$$

$$\alpha_2(1) = \alpha_1(1) \cdot 0.1 \cdot 0.1 = 4 \cdot 10^{-3}$$

$$\alpha_2(2) = [\alpha_1(1) \cdot 0.8 + \alpha_1(2) \cdot 0.1] \cdot 0.8 = 2.6 \cdot 10^{-1}$$

$$\alpha_2(3) = [\alpha_1(1) \cdot 0.1 + \alpha_1(2) \cdot 0.9] \cdot 0.5 = 4.25 \cdot 10^{-2}$$

$$\alpha_3(1) = \alpha_2(1) \cdot 0.1 \cdot 0.1 = 4 \cdot 10^{-5}$$

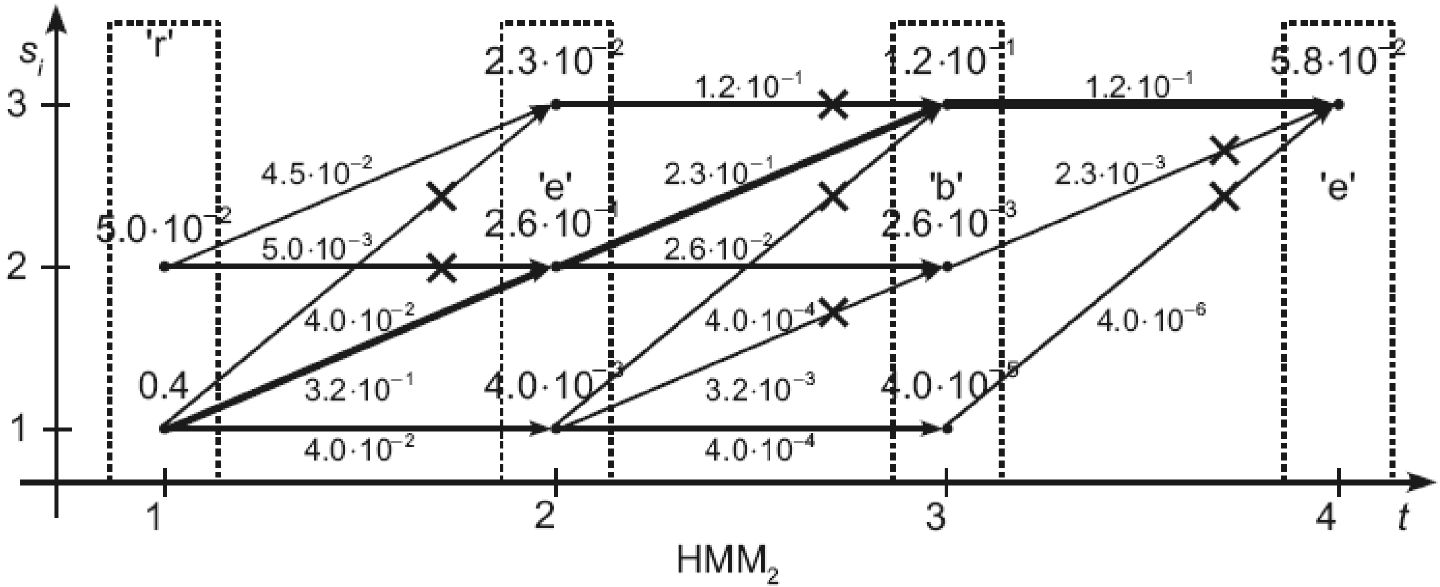
$$\alpha_3(2) = [\alpha_2(1) \cdot 0.8 + \alpha_2(2) \cdot 0.1] \cdot 0.1 = 2.92 \cdot 10^{-3}$$

$$\alpha_3(3) = [\alpha_2(1) \cdot 0.1 + \alpha_2(2) \cdot 0.9 + \alpha_2(3)] \cdot 0.5 = 1.38 \cdot 10^{-1}$$

$$\alpha_4(3) = [\alpha_3(1) \cdot 0.1 + \alpha_3(2) \cdot 0.9 + \alpha_3(3)] \cdot 0.5 = 7.05 \cdot 10^{-2}$$

$$P(o^1 \mid \lambda_2) = 7.05 \cdot 10^{-2}$$

Optimale Folge – Viterbi Alg.



Optimale Folge

$$o^1 = (r, e, b, e)$$

(r,e,b,e)

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

Optimale Folge: 1 2 3 3

Vergleich

$$o^1 = (r, e, b, e)$$

$$p(o^1 \mid \lambda_1)$$

(b,e,e,r,e)

$$p(o^1 \mid \lambda_2)$$

(r,e,b,e)

$$P(o^1 \mid \lambda_1) = 7.34 \cdot 10^{-4}$$

$$P(o^1 \mid \lambda_2) = 7.05 \cdot 10^{-2}$$

$$p(o^1 \mid \lambda_2) > p(o^1 \mid \lambda_1)$$

Bemerkung

$$o^1 = (r, e, b, e)$$

$$P(o^1 \mid \lambda_2) = 7.05 \cdot 10^{-2}$$

Warum benutzt man kein HMM für die Klasse $k = (r, e, b, e)$

mit $P(k \mid \lambda_2) = P(o^1 \mid \lambda_2) = 1$

Alle anderen Beobachtungen treten dann mit Wahrscheinlichkeit 0 auf, z.B auch:

$$o^3 = (r, e, e, b, e)$$

Für die Klassifikation nicht günstig.

Auftrittswahr. für Beobachtung 2

$$o^2 = (b, e, e, r, e)$$

$$p(o^2 \mid \lambda_1)$$ → (b,e,e,r,e)

$$p(o^2 \mid \lambda_2)$$ → (r,e,b,e)

In beiden Fällen wollen wir zum Zeitpunkt 5 im Endzustand 3 sein.

Wir erwarten: $p(o^2 \mid \lambda_2) < p(o^2 \mid \lambda_1)$

Berechnung mit Vorwärtsalgorithmus

$$o^2 = (b, e, e, r, e)$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

b e r

$$\alpha_1(1) = 1 \cdot 0.8 = 0.8$$

$$\alpha_2(1) = \alpha_1(1) \cdot 0.1 \cdot 0.1 = 8 \cdot 10^{-3}$$

$$\alpha_2(2) = \alpha_1(1) \cdot 0.9 \cdot 0.8 = 5.76 \cdot 10^{-1}$$

$$\alpha_3(1) = \alpha_2(1) \cdot 0.1 \cdot 0.1 = 8 \cdot 10^{-5}$$

$$\alpha_3(2) = [\alpha_2(1) \cdot 0.9 + \alpha_2(2) \cdot 0.2] \cdot 0.8 = 9.79 \cdot 10^{-2}$$

$$\alpha_3(3) = \alpha_2(2) \cdot 0.8 \cdot 0.6 = 2.76 \cdot 10^{-1}$$

$$\alpha_4(1) = \alpha_3(1) \cdot 0.1 \cdot 0.1 = 8 \cdot 10^{-7}$$

$$\alpha_4(2) = [\alpha_3(1) \cdot 0.9 + \alpha_3(2) \cdot 0.2] \cdot 0.1 = 1.97 \cdot 10^{-3}$$

$$\alpha_4(3) = [\alpha_3(2) \cdot 0.8 + \alpha_3(3)] \cdot 0.4 = 1.42 \cdot 10^{-1}$$

$$\alpha_5(3) = [\alpha_4(2) \cdot 0.8 + \alpha_4(3)] \cdot 0.6 = 8.61 \cdot 10^{-2}$$

$$p(o^2 \mid \lambda_1)$$

Berechnung mit Vorwärtsalgorithmus

$$o^2 = (b, e, e, r, e)$$

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

$$\alpha_1(1) = 0.5 \cdot 0.1 = 5 \cdot 10^{-2}$$
$$\alpha_1(2) = 0.5 \cdot 0.1 = 5 \cdot 10^{-2}$$
$$\alpha_2(1) = \alpha_1(1) \cdot 0.1 \cdot 0.1 = 5 \cdot 10^{-4}$$
$$\alpha_2(2) = [\alpha_1(1) \cdot 0.8 + \alpha_1(2) \cdot 0.1] \cdot 0.8 = 3.6 \cdot 10^{-2}$$
$$\alpha_2(3) = [\alpha_1(1) \cdot 0.1 + \alpha_1(2) \cdot 0.9] \cdot 0.5 = 2.5 \cdot 10^{-2}$$
$$\alpha_3(1) = \alpha_2(1) \cdot 0.1 \cdot 0.1 = 5 \cdot 10^{-6}$$
$$\alpha_3(2) = [\alpha_2(1) \cdot 0.8 + \alpha_2(2) \cdot 0.1] \cdot 0.8 = 3.2 \cdot 10^{-3}$$
$$\alpha_3(3) = [\alpha_2(1) \cdot 0.1 + \alpha_2(2) \cdot 0.9 + \alpha_2(3)] \cdot 0.5 = 2.87 \cdot 10^{-2}$$
$$\alpha_4(1) = \alpha_3(1) \cdot 0.1 \cdot 0.8 = 4 \cdot 10^{-7}$$
$$\alpha_4(2) = [\alpha_3(1) \cdot 0.8 + \alpha_3(2) \cdot 0.1] \cdot 0.1 = 3.24 \cdot 10^{-5}$$
$$\alpha_5(3) = [\alpha_4(1) \cdot 0.1 + \alpha_4(2) \cdot 0.9] \cdot 0.5 = 1.46 \cdot 10^{-5}$$

$$p(o^2 \mid \lambda_2)$$

Vergleich

$$o^2 = (b, e, e, r, e)$$

$$p(o^2 \mid \lambda_1)$$ → (b,e,e,r,e)

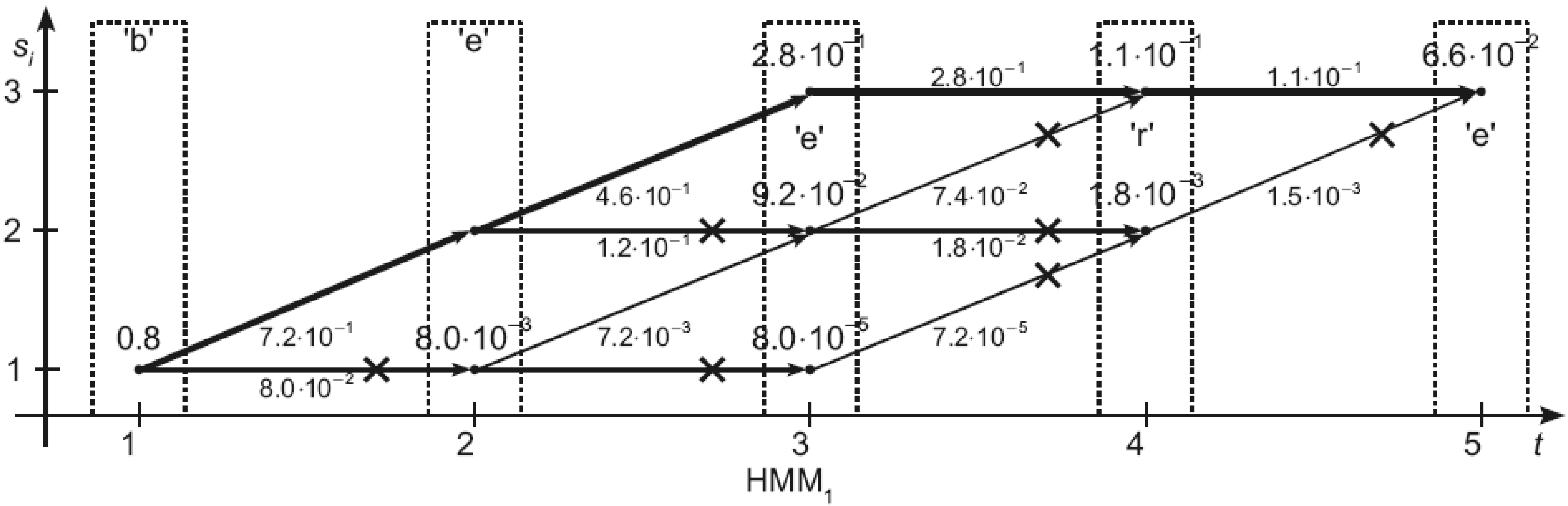
$$p(o^2 \mid \lambda_2)$$ → (r,e,b,e)

$$P(o^2 \mid \lambda_1) = 8.61 \cdot 10^{-2}$$

$$P(o^2 \mid \lambda_2) = 1.46 \cdot 10^{-5}$$

$$p(o^2 \mid \lambda_2) < p(o^2 \mid \lambda_1)$$

Viterbi – Algorithmus HMM1



Optimale Folge

$$o^2 = (b, e, e, r, e)$$

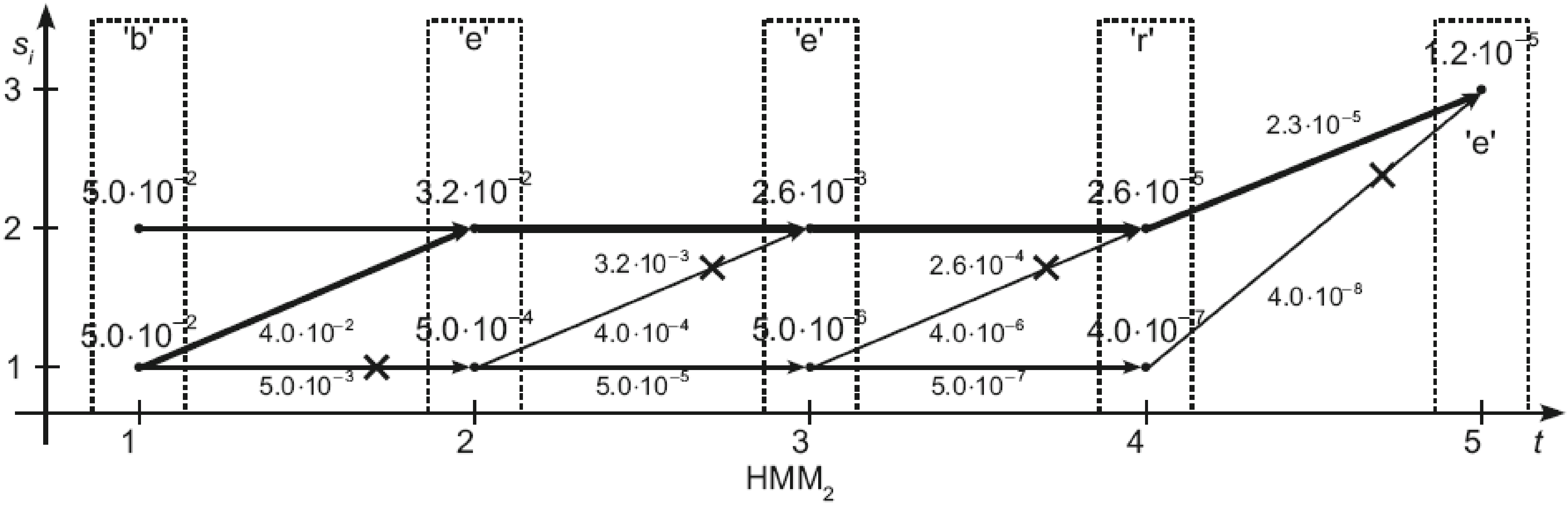
(b,e,e,r,e)

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

Optimale Folge: 1 2 3 3 3

Viterbi – Algorithmus HMM2



Optimale Folge

$$o^2 = (b, e, e, r, e)$$

(r,e,b,e)

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

Optimale Folge: 1 2 2 2 3

Beobachtung 3

$$o^3 = (r, e, e, b, e)$$

$$p(o^3 \mid \lambda_1)$$

(b,e,e,r,e)

$$p(o^3 \mid \lambda_2)$$

(r,e,b,e)

besser

$$p(o^3 \mid \lambda_1) = 1.18 \cdot 10^{-4}$$

$$p(o^3 \mid \lambda_2) = 4.00 \cdot 10^{-2}$$

Vorwärtsalgorithmus – HMM 1

$$\alpha_1(1) = 1 \cdot 0.1 = 0.1$$

$$\alpha_2(1) = \alpha_1(1) \cdot 0.1 \cdot 0.1 = 1 \cdot 10^{-3}$$

$$\alpha_2(2) = \alpha_1(1) \cdot 0.9 \cdot 0.8 = 7.2 \cdot 10^{-2}$$

$$\alpha_3(1) = \alpha_2(1) \cdot 0.1 \cdot 0.1 = 1 \cdot 10^{-5}$$

$$\alpha_3(2) = [\alpha_2(1) \cdot 0.9 + \alpha_2(2) \cdot 0.2] \cdot 0.8 = 1.22 \cdot 10^{-2}$$

$$\alpha_3(3) = \alpha_2(2) \cdot 0.8 \cdot 0.6 = 3.46 \cdot 10^{-2}$$

$$\alpha_4(1) = \alpha_3(1) \cdot 0.1 \cdot 0.8 = 8 \cdot 10^{-7}$$

$$\alpha_4(2) = [\alpha_3(1) \cdot 0.9 + \alpha_3(2) \cdot 0.2] \cdot 0.1 = 2.46 \cdot 10^{-4}$$

$$\alpha_5(3) = \alpha_4(2) \cdot 0.8 \cdot 0.6 = 1.18 \cdot 10^{-4}$$

$$p(o^3 \mid \lambda_1) = 1.18 \cdot 10^{-4}$$

Vorwärtsalgorithmus – HMM 2

$$\alpha_1(1) = 0.5 \cdot 0.8 = 0.4$$

$$\alpha_1(2) = 0.5 \cdot 0.1 = 5 \cdot 10^{-2}$$

$$\alpha_2(1) = \alpha_1(1) \cdot 0.1 \cdot 0.1 = 4 \cdot 10^{-3}$$

$$\alpha_2(2) = [\alpha_1(1) \cdot 0.8 + \alpha_1(2) \cdot 0.1] \cdot 0.8 = 2.6 \cdot 10^{-1}$$

$$\alpha_2(3) = [\alpha_1(1) \cdot 0.1 + \alpha_1(2) \cdot 0.9] \cdot 0.5 = 4.25 \cdot 10^{-2}$$

$$\alpha_3(1) = \alpha_2(1) \cdot 0.1 \cdot 0.1 = 4 \cdot 10^{-5}$$

$$\alpha_3(2) = [\alpha_2(1) \cdot 0.8 + \alpha_2(2) \cdot 0.1] \cdot 0.8 = 2.34 \cdot 10^{-2}$$

$$\alpha_3(3) = [\alpha_2(1) \cdot 0.1 + \alpha_2(2) \cdot 0.9 + \alpha_2(3)] \cdot 0.5 = 1.38 \cdot 10^{-1}$$

$$\alpha_4(1) = \alpha_3(1) \cdot 0.1 \cdot 0.1 = 4 \cdot 10^{-7}$$

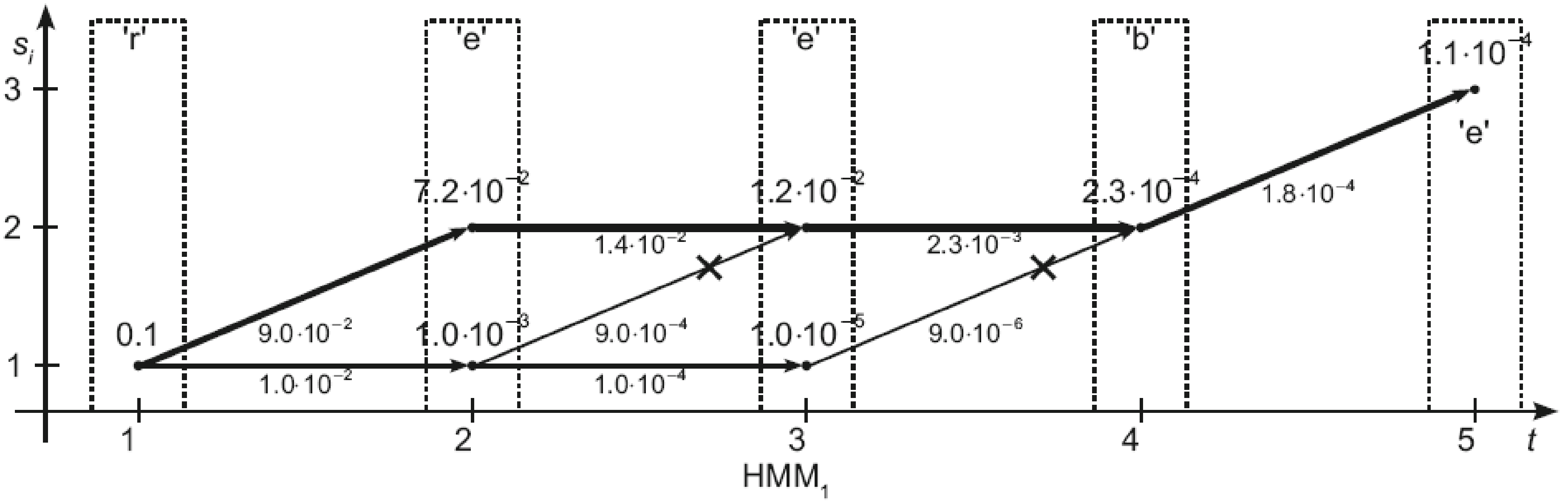
$$\alpha_4(2) = [\alpha_3(1) \cdot 0.8 + \alpha_3(2) \cdot 0.1] \cdot 0.1 = 2.37 \cdot 10^{-4}$$

$$\alpha_4(3) = [\alpha_3(1) \cdot 0.1 + \alpha_3(2) \cdot 0.9 + \alpha_3(3)] \cdot 0.5 = 7.97 \cdot 10^{-2}$$

$$\alpha_5(3) = [\alpha_4(1) \cdot 0.1 + \alpha_4(2) \cdot 0.9 + \alpha_4(3)] \cdot 0.5 = 4.00 \cdot 10^{-2}$$

$$p(o^3 \mid \lambda_2) = 4.00 \cdot 10^{-2}$$

Viterbi – HMM 1



Optimale Folge

$$o^3 = (r, e, e, b, e)$$

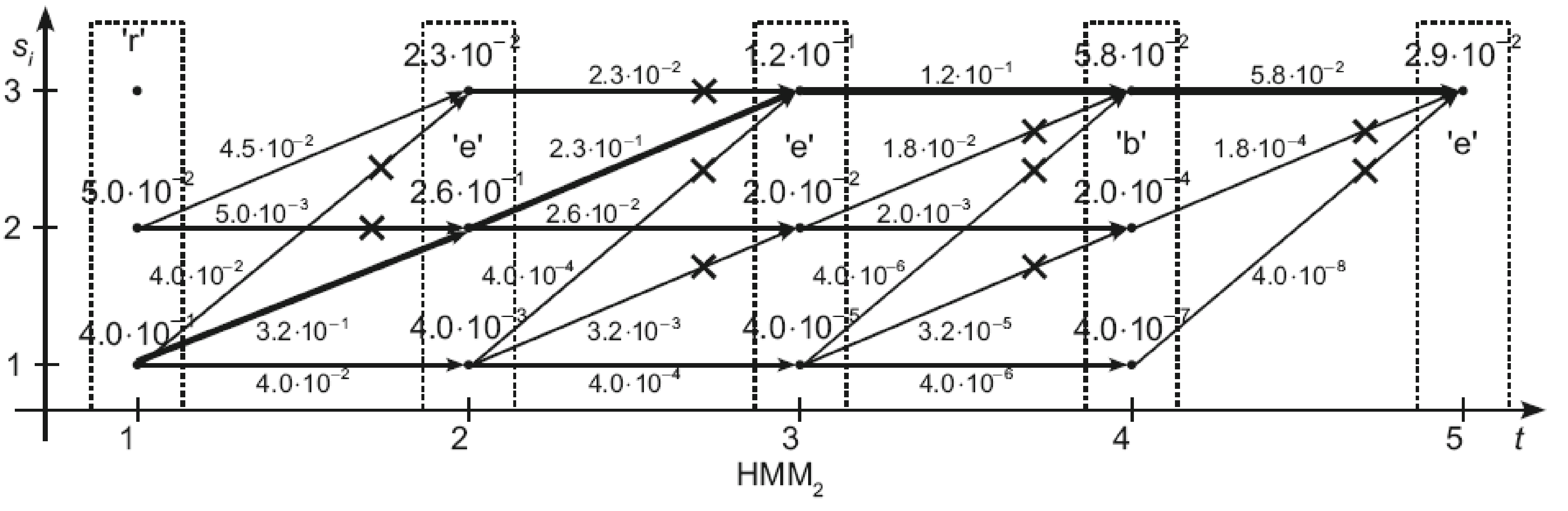
(b,e,e,r,e)

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

Optimale Folge: 1 2 2 2 3

Viterbi – HMM 2



Optimale Folge

$$o^3 = (r, e, e, b, e)$$

(r,e,b,e)

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

Optimale Folge: 1 2 3 3 3

Beobachtung 4

$$o^4 = (b, e, r, e)$$

(b,e,e,r,e)

$$p(o^4 \mid \lambda_1)$$

besser

(r,e,b,e)

$$p(o^4 \mid \lambda_2)$$

$$p(o^4 \mid \lambda_1) = 1.16 \cdot 10^{-1}$$

$$p(o^4 \mid \lambda_2) = 1.82 \cdot 10^{-4}$$

Vorwärtsalgorithmus – HMM 1

$$\alpha_1(1) = 1 \cdot 0.8 = 0.8$$

$$\alpha_2(1) = \alpha_1(1) \cdot 0.1 \cdot 0.1 = 8 \cdot 10^{-3}$$

$$\alpha_2(2) = \alpha_1(1) \cdot 0.9 \cdot 0.8 = 5.76 \cdot 10^{-1}$$

$$\alpha_3(1) = \alpha_2(1) \cdot 0.1 \cdot 0.1 = 8 \cdot 10^{-5}$$

$$\alpha_3(2) = [\alpha_2(1) \cdot 0.9 + \alpha_2(2) \cdot 0.2] \cdot 0.1 = 1.22 \cdot 10^{-2}$$

$$\alpha_3(3) = \alpha_2(2) \cdot 0.8 \cdot 0.4 = 1.84 \cdot 10^{-1}$$

$$\alpha_4(3) = [\alpha_3(2) \cdot 0.8 + \alpha_3(3)] \cdot 0.6 = 1.16 \cdot 10^{-1}$$

$$p(o^4 \mid \lambda_1) = 1.16 \cdot 10^{-1}$$

Vorwärtsalgorithmus – HMM 2

$$\alpha_1(1) = 0.5 \cdot 0.1 = 5 \cdot 10^{-2}$$

$$\alpha_1(2) = 0.5 \cdot 0.1 = 5 \cdot 10^{-2}$$

$$\alpha_2(1) = \alpha_1(1) \cdot 0.1 \cdot 0.1 = 5 \cdot 10^{-4}$$

$$\alpha_2(2) = [\alpha_1(1) \cdot 0.8 + \alpha_1(2) \cdot 0.1] \cdot 0.8 = 3.6 \cdot 10^{-2}$$

$$\alpha_2(3) = [\alpha_1(1) \cdot 0.1 + \alpha_1(2) \cdot 0.9] \cdot 0.5 = 2.5 \cdot 10^{-2}$$

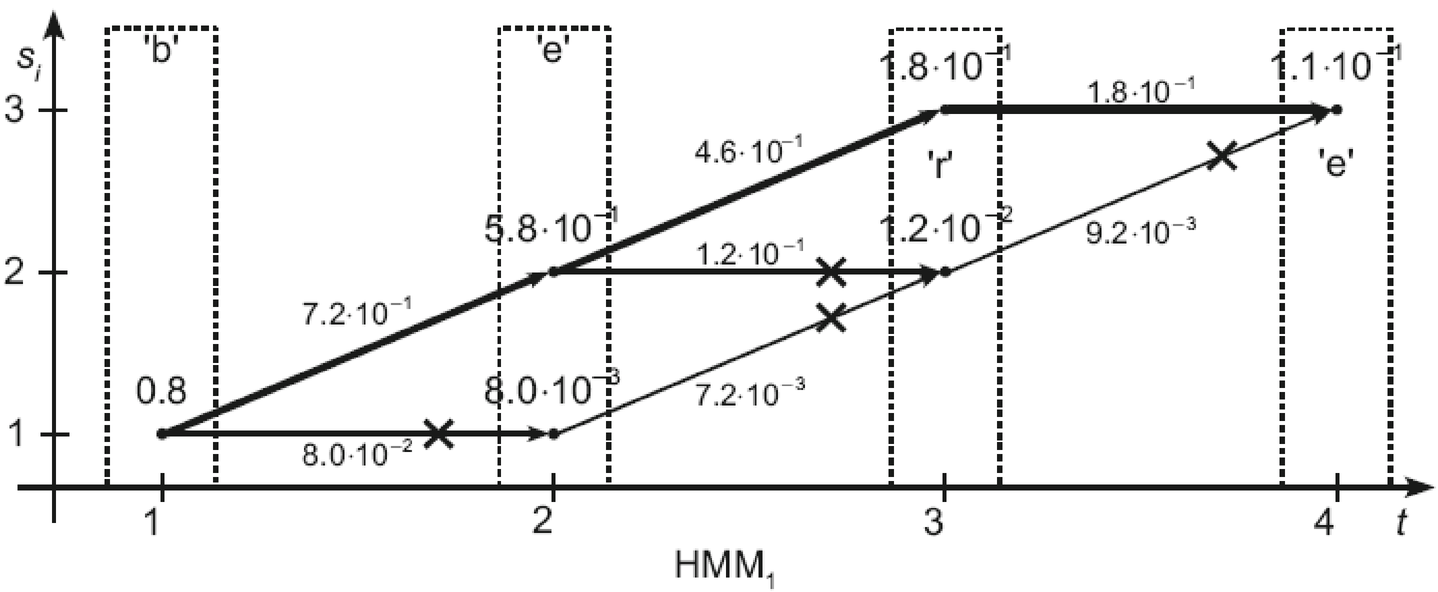
$$\alpha_3(1) = \alpha_2(1) \cdot 0.1 \cdot 0.8 = 4 \cdot 10^{-5}$$

$$\alpha_3(2) = [\alpha_2(1) \cdot 0.8 + \alpha_2(2) \cdot 0.1] \cdot 0.1 = 4 \cdot 10^{-4}$$

$$\alpha_4(3) = [\alpha_3(1) \cdot 0.1 + \alpha_3(2) \cdot 0.9] \cdot 0.5 = 1.82 \cdot 10^{-4}$$

$$p(o^4 \mid \lambda_2) = 1.82 \cdot 10^{-4}$$

Viterbi – HMM 1



Optimale Folge

$$o^4 = (b, e, r, e)$$

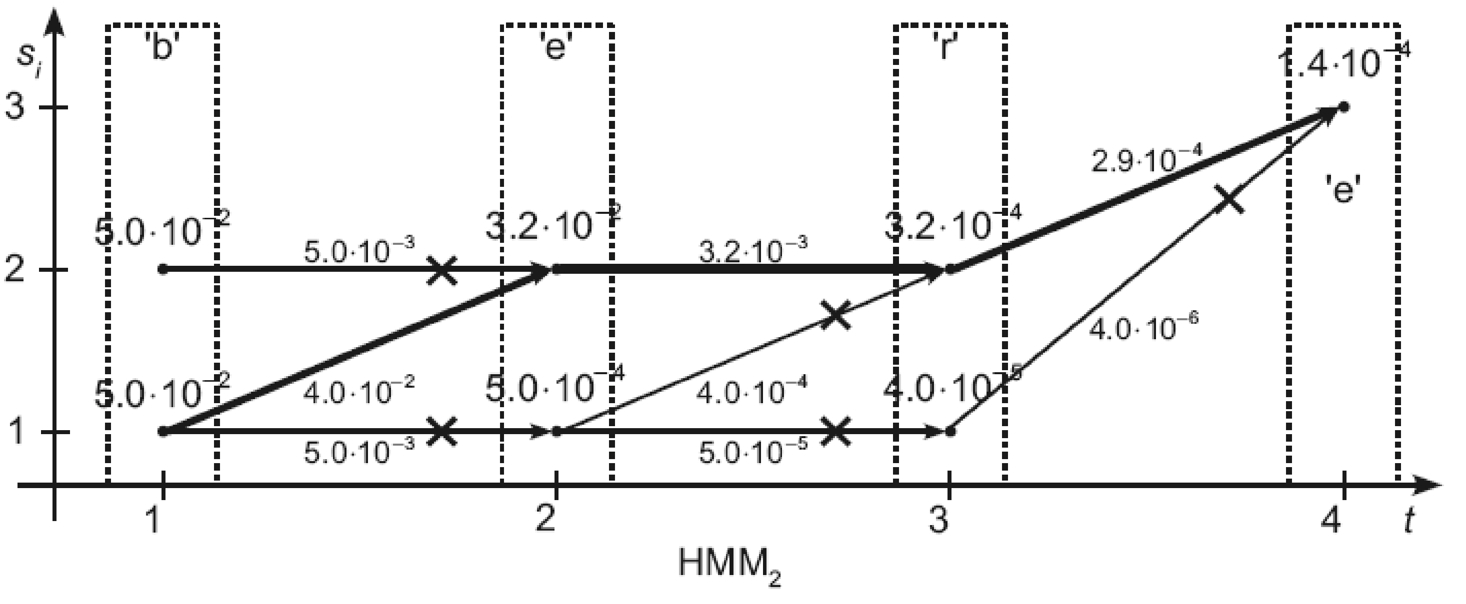
(b,e,e,r,e)

b e r

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{A}_1 = \begin{pmatrix} 0.1 & 0.9 & 0 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.6 & 0.4 \end{pmatrix}$$

Optimale Folge: 1 2 3 3

Viterbi – HMM 2



Optimale Folge

$$o^4 = (b, e, r, e)$$

(r,e,b,e)

b e r

$$\mathbf{e}_2 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 0.1 & 0.8 & 0.1 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_2 = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

Optimale Folge: 1 2 2 3

Alle Ergebnisse

HMM 1	HMM 2
(b,e,e,r,e)	(r,e,b,e)

$$o^1 = (r, e, b, e)$$
$$o^2 = (b, e, e, r, e)$$
$$o^3 = (r, e, e, b, e)$$
$$o^4 = (b, e, r, e)$$

$$P(o^1 \mid \lambda_1) = 7.34 \cdot 10^{-4}$$
$$P(o^2 \mid \lambda_1) = 8.61 \cdot 10^{-2}$$
$$p(o^3 \mid \lambda_1) = 1.18 \cdot 10^{-4}$$
$$p(o^4 \mid \lambda_1) = 1.16 \cdot 10^{-1}$$

$$P(o^1 \mid \lambda_2) = 7.05 \cdot 10^{-2}$$
$$P(o^2 \mid \lambda_2) = 1.46 \cdot 10^{-5}$$
$$p(o^3 \mid \lambda_2) = 4.00 \cdot 10^{-2}$$
$$p(o^4 \mid \lambda_2) = 1.82 \cdot 10^{-4}$$

Quelle

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  Mensch – Maschine – Kommunikation
  Springer, 2010