

Übung 5

20.12.2011

Beispiel – Regeln

$$\left[\begin{array}{l} CAT : S \\ HEAD : \boxed{1} \end{array} \right] \rightarrow \left[\begin{array}{l} CAT : NP \\ HEAD : \boxed{2} \end{array} \right] \left[\begin{array}{l} CAT : VP \\ HEAD : \boxed{1} \left[\begin{array}{l} AGR \boxed{2} \end{array} \right] \end{array} \right]$$

(Regel: $S \rightarrow NP \ VP$)

$$\left[\begin{array}{l} CAT : VP \\ HEAD : \boxed{3} \end{array} \right] \rightarrow \left[\begin{array}{l} CAT : V \\ HEAD : \boxed{3} \end{array} \right] \left[\begin{array}{l} CAT : NP \end{array} \right]$$

(Regel: $VP \rightarrow V \ NP$)

Beispiel – Regeln

$$\left[\begin{array}{l} CAT : NP \\ HEAD : \boxed{4} \end{array} \right] \rightarrow \left[\begin{array}{l} CAT : N \\ HEAD : \boxed{4} \end{array} \right]$$

(Regel: $NP \rightarrow N$)

$$\left[\begin{array}{l} CAT : NP \\ HEAD : \boxed{5} \end{array} \right] \rightarrow [CAT : ART] \left[\begin{array}{l} CAT : N \\ HEAD : \boxed{5} \end{array} \right]$$

(Regel: $NP \rightarrow ART \ N$)

Beispiel – Initialisierung der Chart

$$\left[\begin{array}{l} \text{CAT} : S \\ \text{HEAD} : \boxed{1} \end{array} \right] \rightarrow \left[\begin{array}{l} \text{CAT} : NP \\ \text{HEAD} : \boxed{2} \end{array} \right] \left[\begin{array}{l} \text{CAT} : VP \\ \text{HEAD} : \boxed{1} \left[\text{AGR} \boxed{2} \right] \end{array} \right]$$

(Regel: $S \rightarrow NP \ VP$)

$$\left[0, 0, \left[\begin{array}{l} \text{CAT} : S \\ \text{HEAD} : \boxed{6} \end{array} \right], \varepsilon, \left[\begin{array}{l} \text{CAT} : NP \\ \text{HEAD} : \boxed{7} \end{array} \right] \left[\begin{array}{l} \text{CAT} : VP \\ \text{HEAD} : \boxed{6} \end{array} \right], \left\{ \left[\boxed{6} \left[\text{AGR} \boxed{7} \right] \right] \right\} \right]$$

Beispiel – Expand

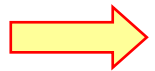
$$\left[0, 0, \left[\begin{array}{l} \text{CAT} : NP \\ \text{HEAD} : \boxed{8} \end{array} \right], \varepsilon, \left[\begin{array}{l} \text{CAT} : N \\ \text{HEAD} : \boxed{8} \end{array} \right], \{ \} \right]$$

$$\left[0, 0, \left[\begin{array}{l} \text{CAT} : NP \\ \text{HEAD} : \boxed{9} \end{array} \right], \varepsilon, [\text{CAT} : \text{ART}] \quad \left[\begin{array}{l} \text{CAT} : N \\ \text{HEAD} : \boxed{9} \end{array} \right], \{ \} \right]$$

Beispiel - Complete

$$\left[2, 2, \left[\begin{array}{l} \text{CAT} : \text{VP} \\ \text{HEAD} : \boxed{10} \end{array} \right], \varepsilon, \left[\begin{array}{l} \text{CAT} : \text{V} \\ \text{HEAD} : \boxed{10} \end{array} \right] \left[\text{CAT} : \text{NP} \right], \{ \} \right]$$

$$\left[2, 3, \left[\begin{array}{l} \text{CAT} : \text{V} \\ \text{HEAD} : \left[\begin{array}{l} \text{PER} : 3 \\ \text{TMP} : \text{praes} \\ \text{NUM} : \text{sing} \end{array} \right] \end{array} \right], \text{schlaeft}, \varepsilon, \{ \} \right]$$



$$\left[2, 3, \left[\begin{array}{l} \text{CAT} : \text{VP} \\ \text{HEAD} : \boxed{11} \end{array} \right], \left[\begin{array}{l} \text{CAT} : \text{V} \\ \text{HEAD} : \boxed{11} \end{array} \right], \left[\text{CAT} : \text{NP} \right], \left\{ \left[\begin{array}{l} \boxed{11} \left[\begin{array}{l} \text{PER} : 3 \\ \text{TMP} : \text{praes} \\ \text{NUM} : \text{sing} \end{array} \right] \right] \right\} \right]$$

Logik

1 – Syntax

Konstanten:

peter

johanna

duden

algebra

Prädikatssymbole (Relationen):

schlafen(_)

buch(_)

lesen(_,_)

1 – stellig

2 – stellig



2 – Semantik

Modell (Struktur): $M = \langle A, I \rangle$

$A = \{\text{Peter, Hans, Johanna, Duden, AlgebraII}\}$

$I(\text{peter}) = \text{Peter}$

$I(\text{johanna}) = \text{Johanna}$

$I(\text{duden}) = \text{Duden}$

$I(\text{algebra}) = \text{AlgebraII}$

Konstanten

$\in A$

Hier wird Hans nicht benutzt.

2 – Semantik

$$I(\text{schlafen}) = \{\text{Peter}\}$$

$$I(\text{buch}) = \{\text{Duden, AlgebraII}\}$$

$$\subseteq A^1$$

Relationen

$$I(\text{lesen}) = \{(\text{Peter, Duden}), (\text{Johanna, Duden}), (\text{Hans, AlgebraII})\}$$

Hier wird Hans benötigt.

$$\subseteq A^2$$

3 – Interpretationen

Variablenbelegung: $g(x) = \text{Hans}$
 $g(y) = \text{Duden}$

Interpretation – Konstante:

$$[[\text{johanna}]]^{M,g} = I(\text{johanna}) = \text{Johanna}$$

Interpretation – Variable:

$$[[g]]^{M,g} = g(y) = \text{Duden}$$

3 – Interpretationen

Interpretation – Formel: $\text{lesen}(x, y)$

$$\llbracket \text{lesen}(x, y) \rrbracket^{M,g} = 1 \Leftrightarrow (g(x), g(y)) \in I(\text{lesen})$$

$$(\text{Hans}, \text{Duden}) \notin I(\text{lesen}) \quad \Rightarrow \quad \llbracket \text{lesen}(x, y) \rrbracket^{M,g} = 0 \quad (\text{falsch})$$

andere Variablenbelegung:

$$h(x) = \text{Peter} \quad \Rightarrow \quad \llbracket \text{lesen}(x, y) \rrbracket^{M,h} = 1 \quad (\text{wahr})$$

$$h(y) = \text{Duden} \quad (\text{Peter}, \text{Duden}) \in I(\text{lesen})$$

4 – Typenlogik – Konstanten

Typ e

peter _{e} johanna _{e} duden _{e} algebra _{e}

Typ $\langle e, t \rangle$


schlafen _{$\langle e, t \rangle$} buch _{$\langle e, t \rangle$}

Typ $\langle e, \langle e, t \rangle \rangle$

lesen _{$\langle e, \langle e, t \rangle \rangle$}

4 – Typenlogik – Variablen

x_e y_e

$P_{\langle e,t \rangle}$ $Q_{\langle e,t \rangle}$  neu

4 – Typenlogik – Terme

$x_e = \text{peter}_e$

Typ t (Formel)

$\text{lesen}_{\langle e, \langle e, t \rangle \rangle}(\text{duden}_e)$

Typ $\langle e, t \rangle$

$\text{lesen}_{\langle e, \langle e, t \rangle \rangle}(\text{duden}_e)(x_e)$

Typ t

$x_e = \text{peter}_e \wedge \text{lesen}_{\langle e, \langle e, t \rangle \rangle}(\text{duden}_e)(x_e)$

Typ t

$\forall P_{\langle e, t \rangle} (P_{\langle e, t \rangle}(\text{peter}_e) \rightarrow P_{\langle e, t \rangle}(\text{johanna}_e))$

Typ t

5 – Typenlogik – Semantik

$$M = \langle A, V, D \rangle \quad V : K_\sigma \rightarrow D_\sigma$$

$$D = \{D_\sigma : \sigma \in T, D_\sigma \text{ - beliebige Menge}\}$$

$$D_e = A \quad D_t = \{0,1\} \quad D_{\langle\sigma,\tau\rangle} = D_\tau^{D_\sigma}$$

$$D_{\langle e,t\rangle} = \{f : D_e \rightarrow D_t = \{0,1\}\}$$

$$D_e = A = \{\text{Peter, Hans, Johanna, Duden, AlgebraII}\}$$

5 – Typenlogik – Semantik

$$V : K_e \rightarrow D_e$$

$$V(\text{peter}_e) = \text{Peter}$$

$$V(\text{johanna}_e) = \text{Johanna}$$

$$V(\text{duden}_e) = \text{Duden}$$

$$V(\text{algebra}_e) = \text{AlgebraII}$$

5 – Typenlogik – Semantik

$$V : K_{\langle e,t \rangle} \rightarrow D_{\langle e,t \rangle}$$

$$V(\text{schlafen}_{\langle e,t \rangle}) = f_1 : D_e \rightarrow \{0,1\}$$

$$V(\text{buch}_{\langle e,t \rangle}) = f_2 : D_e \rightarrow \{0,1\}$$

$$f_1 : \text{Peter} \rightarrow 1$$

$$\text{Hans} \rightarrow 0$$

$$\text{Johanna} \rightarrow 0$$

$$\text{Duden} \rightarrow 0$$

$$\text{AlgebraII} \rightarrow 0$$

$$f_2 : \text{Peter} \rightarrow 0$$

$$\text{Hans} \rightarrow 0$$

$$\text{Johanna} \rightarrow 0$$

$$\text{Duden} \rightarrow 1$$

$$\text{AlgebraII} \rightarrow 1$$

Schreibweise: $V(\text{schlafen}_{\langle e,t \rangle}) = \{\text{Peter}\}$ $V(\text{buch}_{\langle e,t \rangle}) = \{\text{Duden}, \text{AlgebraII}\}$

5 – Typenlogik – Semantik

$$V : K_{\langle e, \langle e, t \rangle \rangle} \rightarrow D_{\langle e, \langle e, t \rangle \rangle}$$

$$V(\text{buch}_{\langle e, \langle e, t \rangle \rangle}) = f_3 : D_e \rightarrow D_{\langle e, t \rangle}$$

$$f_3 : \text{Duden} \rightarrow \{\text{Peter, Johanna}\}$$

$$\text{AlgebraII} \rightarrow \{\text{Hans}\}$$

$$\text{Peter} \rightarrow \emptyset$$

$$\text{Johanna} \rightarrow \emptyset$$

$$\text{Hans} \rightarrow \emptyset$$