Übung 4 – Morphologische Operationen

Aufgabe 1

$$X = \{(3,2),(3,3),(2,2),(2,3)\}$$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(0,0)

$$B = \{(-1,-1),(-1,0),(-1,1),(0,-1),(0,0),(0,1),(1,-1),(1,0),(1,1)\}$$

1	1	1
1	1	1
1	1	1

(0,0)

Berechne die Dilation: $X \oplus B$

Dilation

$X, B \subseteq Z^n$

$$X \oplus B = \left\{ d \in Z^n : \exists x \in X, \exists b \in B,\ d = x + b \right\}$$

$$X \oplus B = \bigcup_{b \in B} X_b$$

Aufgabe 1

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	1	1	0	0	0	0
0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$(3,2)+(-1,-1)=(2,1)$
$(3,2)+(-1,0)=(2,2)$
$(3,2)+(-1,1)=(2,3)$

$(3,2)+(0,-1)=(3,1)$
$(3,2)+(0,0)=(3,2)$
$(3,2)+(0,1)=(3,3)$

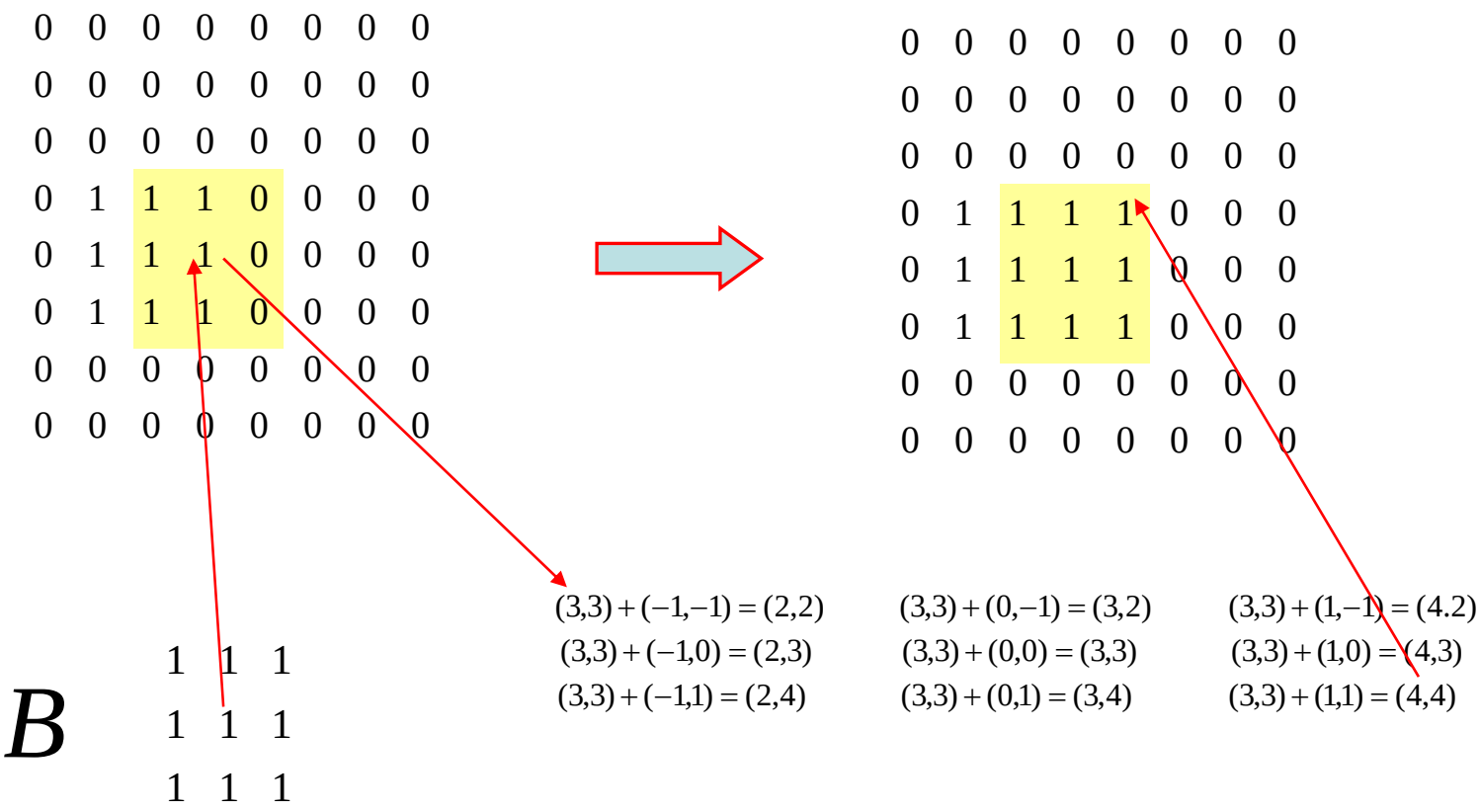
$(3,2)+(1,-1)=(4.1)$
$(3,2)+(1,0)=(4,2)$
$(3,2)+(1,1)=(4,3)$

B

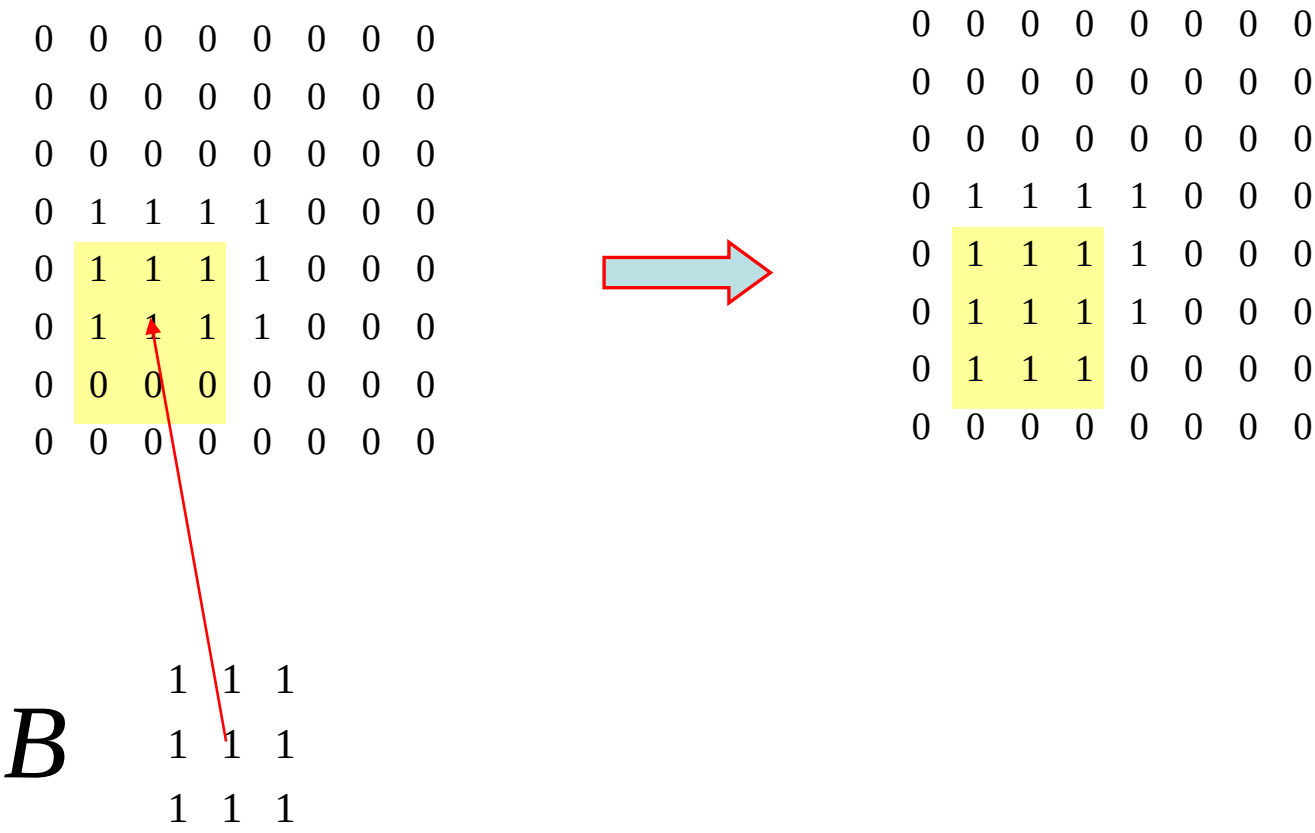
1	1	1
1	1	1
1	1	1

$$X \oplus B = \left\{ d \in Z^n : \exists x \in X, \exists b \in B,\ d = x + b \right\}$$

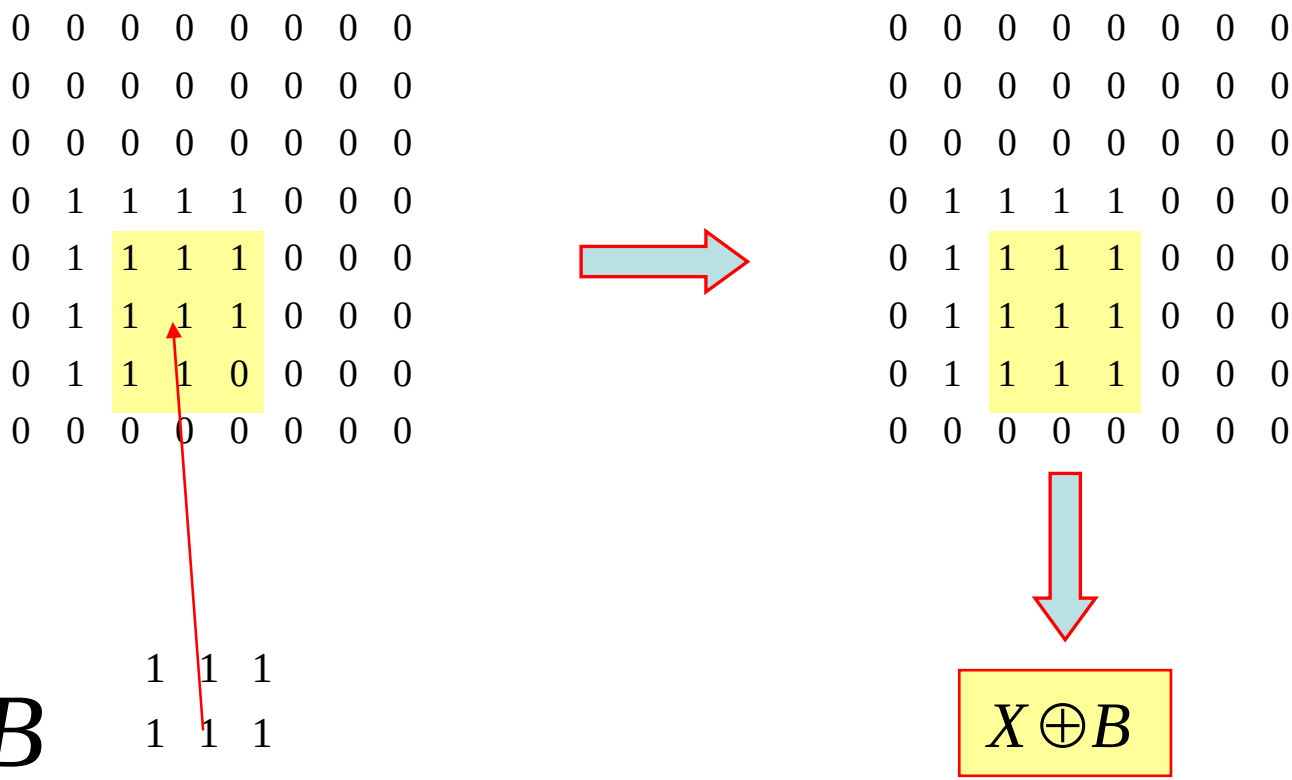
Aufgabe 1


0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 1 1 1 0 0 0 0
0 1 1 1 0 0 0 0
0 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
B
1 1 1
1 1 1
1 1 1
(3,3) + (−1,−1) = (2,2)
(3,3) + (−1,0) = (2,3)
(3,3) + (−1,1) = (2,4)
(3,3) + (0,−1) = (3,2)
(3,3) + (0,0) = (3,3)
(3,3) + (0,1) = (3,4)
(3,3) + (1,−1) = (4.2)
(3,3) + (1,0) = (4,3)
(3,3) + (1,1) = (4,4)

Aufgabe 1


0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
B
1 1 1
1 1 1
1 1 1
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0

Aufgabe 1


0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
B
1 1 1
1 1 1
1 1 1
$X \oplus B$

Aufgabe 1

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$X \oplus B$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0

Aufgabe 1

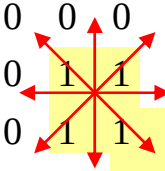
$$X \oplus B = \bigcup_{b \in B} X_b$$

B

1	1	1
1	1	1
1	1	1

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



$X_{(-1,1)}$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0

$X \oplus B$

Aufgabe 2

$$X = \{(3,2),(3,3),(2,2),(2,3)\}$$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(0,0) →

$$B_1 = \{(0,0),(0,1)\}$$

1 1 → (0,0)

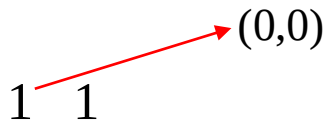
$$B_2 = \{(0,0),(0,-1)\}$$

1 1 → (0,0)

Berechne: $X \oplus B_1$ $X \oplus B_2$

Aufgabe 2

B_1



X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$X \oplus B_1$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Aufgabe 2

B_2

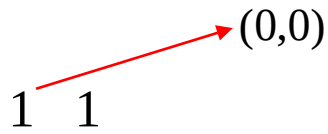
X

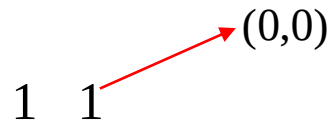
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$X \oplus B_2$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Aufgabe 2





$X \oplus B_1$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$X \oplus B_2$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Aufgabe 3

$$X = \left\{ \begin{matrix} (4,2),(4,3),(4,4),(4,5),(3,2),(3,3),(3,4),(3,5), \\ (2,2),(2,3),(2,4),(2,5),(1,2),(1,3),(1,4),(1,5) \end{matrix} \right\}$$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

$$B = \{(-1,-1),(-1,0),(-1,1),(0,-1),(0,0),(0,1),(1,-1),(1,0),(1,1)\}$$

1	1	1
1	1	1
1	1	1

(0,0)

Berechne die Erosion: $X \ominus B$

Erosion

$X, B \subseteq Z^n$

$$X \ominus B = \left\{ d \in Z^n : \forall b \in B \Rightarrow d + b \in X \right\}$$

$$X \ominus B = \left\{ d \in Z^n : B_d \subseteq X \right\}$$

Aufgabe 3

$$X \ominus B = \left\{ d \in Z^n : B_d \subseteq X \right\}$$

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

B

1	1	1
1	1	1
1	1	1

$\in X \ominus B$

Aufgabe 3

$$X \ominus B = \left\{ d \in Z^n : B_d \subseteq X \right\}$$

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

B

1	1	1
1	1	1
1	1	1

$\in X \ominus B$

Aufgabe 3

$$X \ominus B = \left\{ d \in Z^n : B_d \subseteq X \right\}$$

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

B

1	1	1
1	1	1
1	1	1

$\in X \ominus B$

Aufgabe 3

$$X \ominus B = \left\{ d \in Z^n : B_d \subseteq X \right\}$$

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

B

1	1	1
1	1	1
1	1	1

$\in X \ominus B$

Aufgabe 3

X

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

$X \ominus B$

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Aufgabe 4

Beweisen Sie:

$$X \oplus B = \left\{ d \in Z^n : \tilde{B}_d \cap X \neq \varnothing \right\}$$

Aufgabe 4

$$X \oplus B = \left\{ d \in Z^n : \tilde{B}_d \cap X \neq \varnothing \right\}$$

$$\tilde{B} = \{ -b : b \in B \}$$

$$\tilde{B}_d = \{ d - b : b \in B \}$$

$d \in X \oplus B$ ⟹ $d = x + b$ $\quad x \in X,\ b \in B$

$x = d - b \in \tilde{B}_d$ ⟶ $x \in \tilde{B}_d \cap X \neq \varnothing$

$\tilde{B}_d \cap X \neq \varnothing$ ⟹ $x \in \tilde{B}_d \quad x \in X$

$x = d - b$ ⟹ $d = x + b$ ⟹ $d \in X \oplus B$

Aufgabe 5

X

0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(0,0)

B

1	1	1
1	1	1
1	1	1

Berechnen Sie: $X \setminus (X \ominus B)$

Interpretieren Sie das Ergebnis.

Aufgabe 5

X

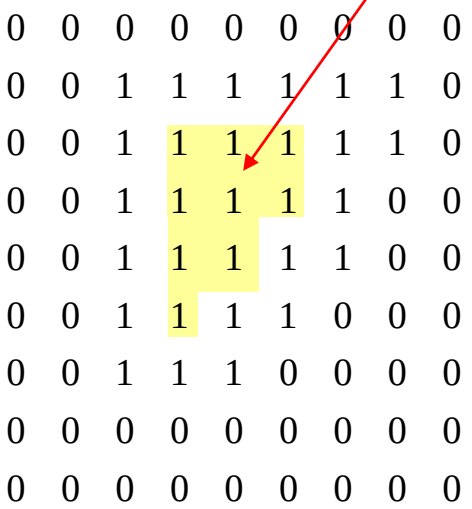
0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

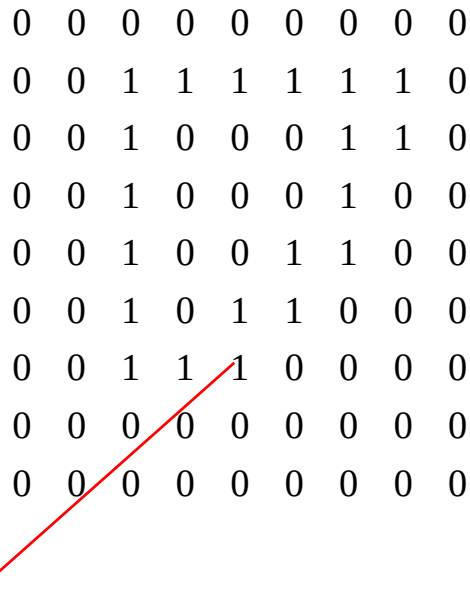
$X \ominus B$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	1	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Aufgabe 5

X $\quad$ $X \ominus B$ $\quad$ $X \setminus (X \ominus B)$





Rand von X

Aufgabe 5_1

X

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

B_1

$$\begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}$$

B_2

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

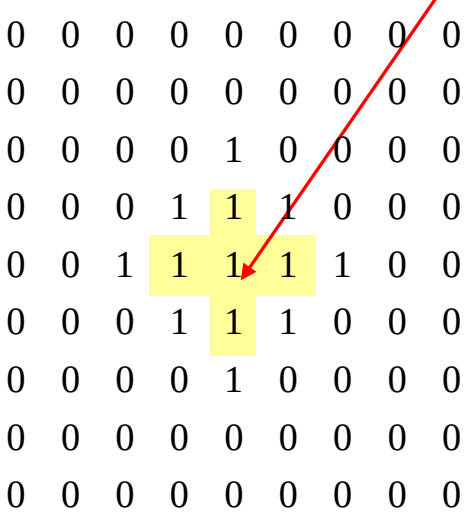
Berechne:

$X \setminus (X \ominus B_1)$

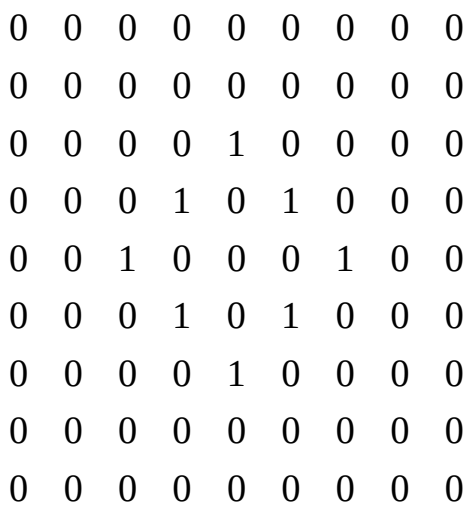
$X \setminus (X \ominus B_2)$

Aufgabe 5_1

X $\qquad$ $X \ominus B_1$ $\qquad$ $X \setminus (X \ominus B_1)$



0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0



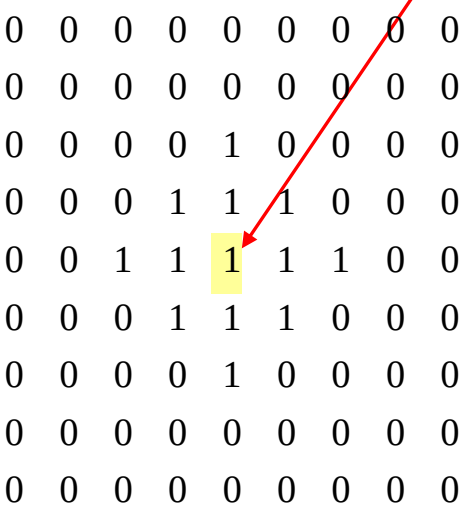
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	0
0	0	1	0	0	0	1	0	0
0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

B_1

0	1	0
1	1	1
0	1	0

Aufgabe 5_1

X $\quad\quad$ $X \ominus B_2$ $\quad\quad$ $X \setminus (X \ominus B_2)$



0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	0	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

B_2

1	1	1
1	1	1
1	1	1

Aufgabe 5_1

X

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

B_1

0	1	0
1	1	1
0	1	0

B_2

1	1	1
1	1	1
1	1	1

$X \setminus (X \ominus B_1)$

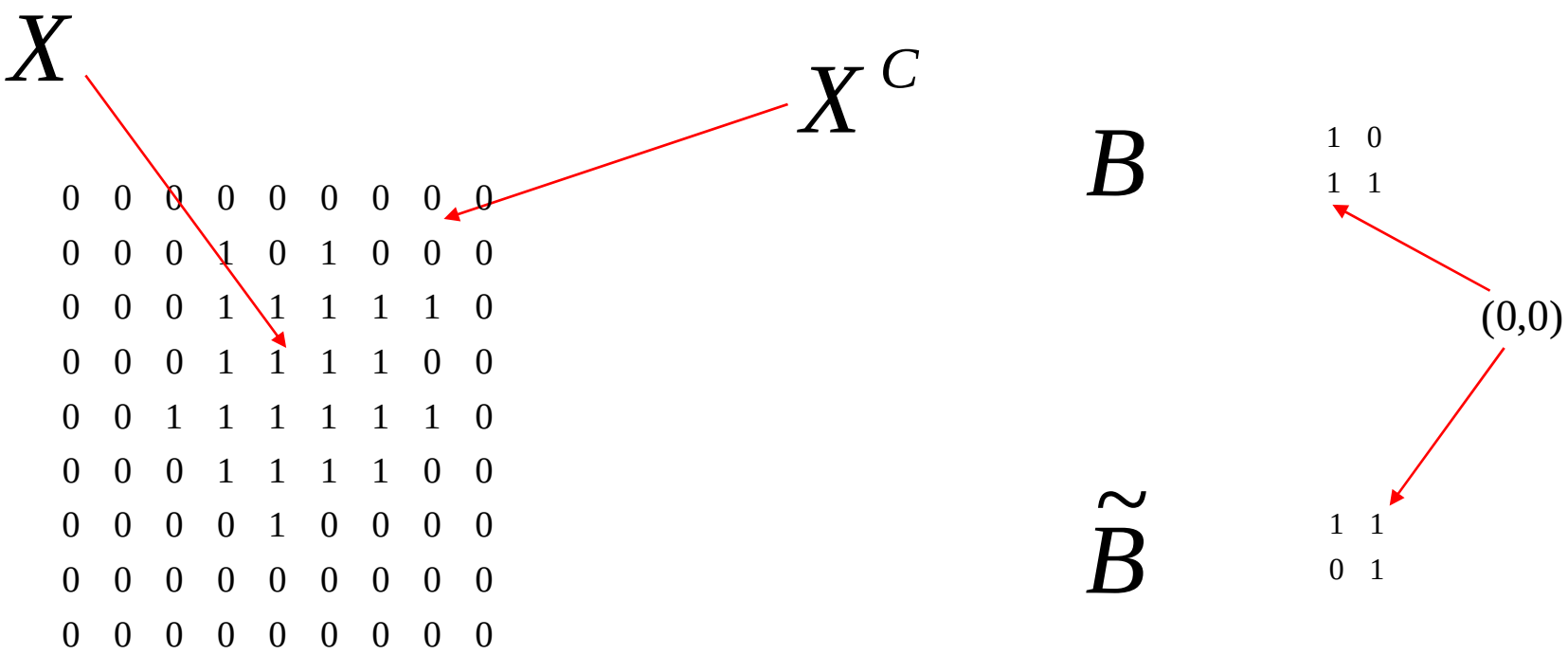
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	0
0	0	1	0	0	0	1	0	0
0	0	0	1	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

$X \setminus (X \ominus B_2)$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	1	0	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Aufgabe 5_2

$$(X \ominus B)^C = X^C \oplus \tilde{B}$$



Aufgabe 5_2

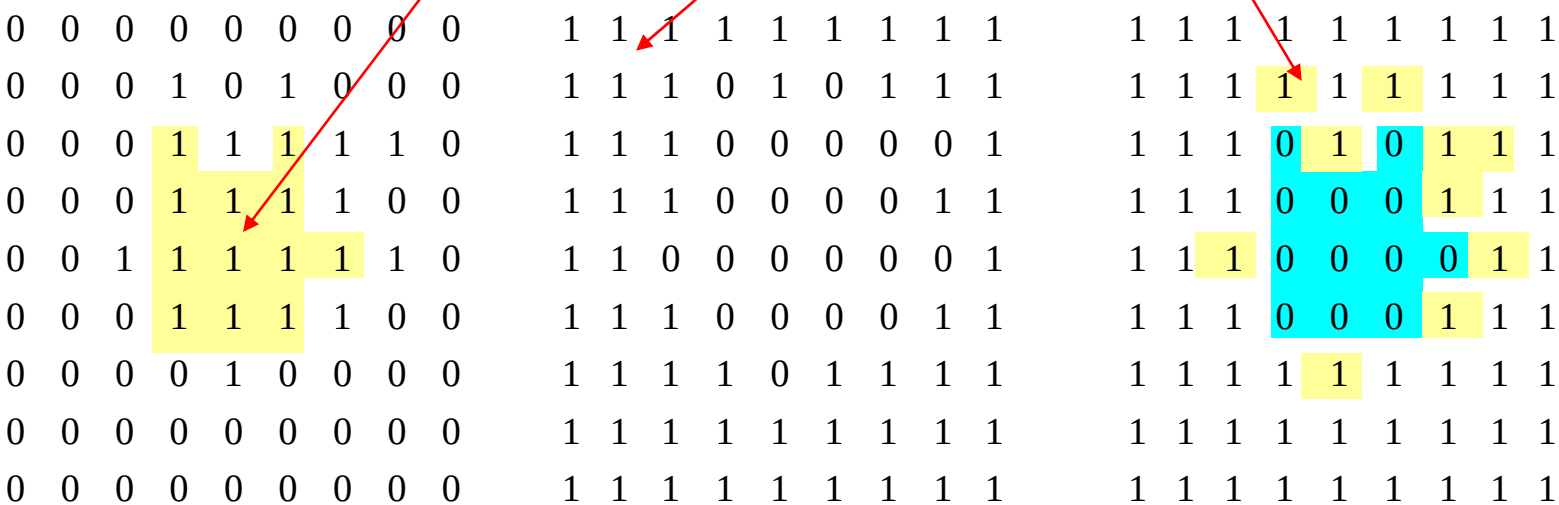
B

1 0
1 1

$$(X \ominus B)^C = X^C \oplus \tilde{B}$$

$\tilde{B}$

1 1
0 1



Aufgabe 6

Welcher Zusammenhang besteht zwischen den Rangfolgeoperationen und den beiden morphologischen Operationen Dilation und Erosion.

Aufgabe 6

$$B \qquad \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

Dilation

$X \oplus B$

Maximaloperation
(3x3)

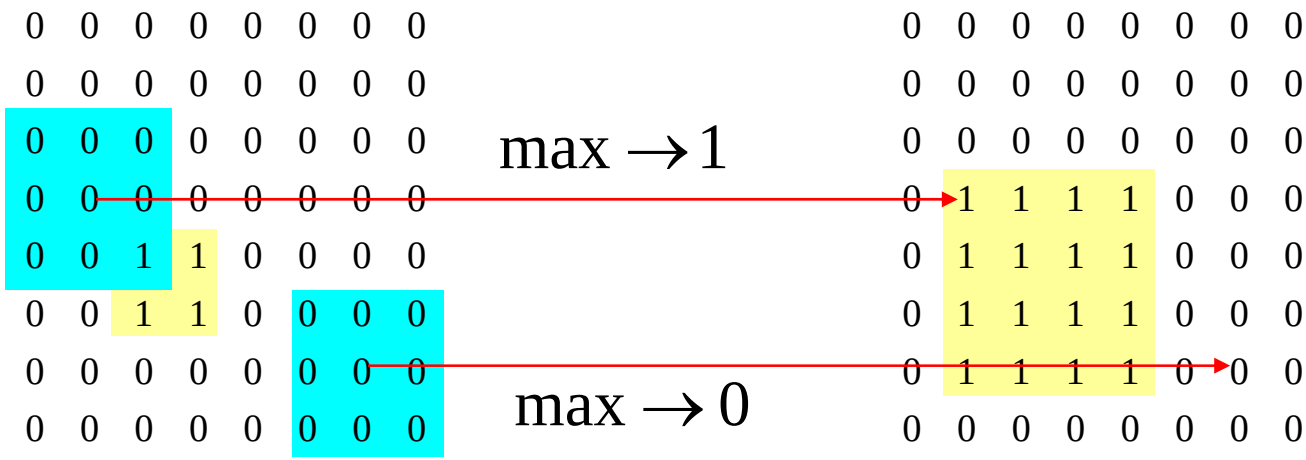
Erosion

$X \ominus B$

Minimaloperation
(3x3)

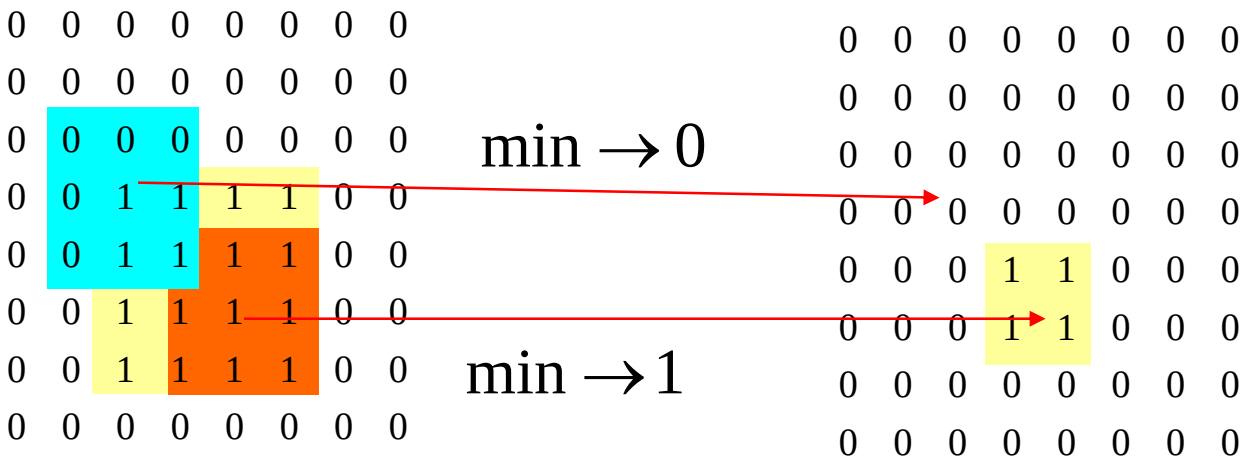
Aufgabe 6
Dilation – Maximaloperation

X $\qquad\qquad$ $X \oplus B$



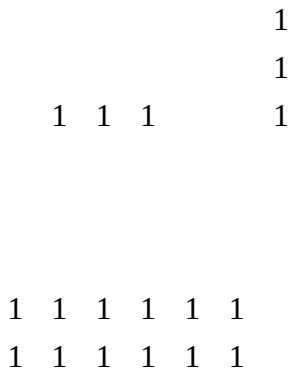
Aufgabe 6
Erosion – Minimaloperation

X $\qquad\qquad$ $X \ominus B$



Aufgabe 7 – Opening

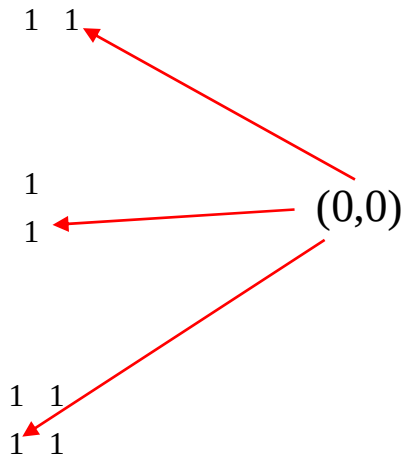
X



B_1

B_2

B_3



Berechne: $X \circ B_1$ $X \circ B_2$ $X \circ B_3$

Vergleiche die Ergebnisse.

Opening - Closing

$X, B \subseteq Z^n$

$$X \circ B = (X \ominus B) \oplus B$$

$$X \bullet B = (X \oplus B) \ominus B$$

Aufgabe 7

B_1

1 1

(0,0)

X

1
1
1 1 1 1

1 1 1 1 1 1
1 1 1 1 1 1

$X \ominus B_1$

–
–
– 1 1 –

– 1 1 1 1 1
– 1 1 1 1 1

Aufgabe 7

B_1

1 1

(0,0)

$X \ominus B_1$

– – –

– 1 1 –

– 1 1 1 1 1

– 1 1 1 1 1

$\oplus B_1$

$X \circ B_1$

1 1 1

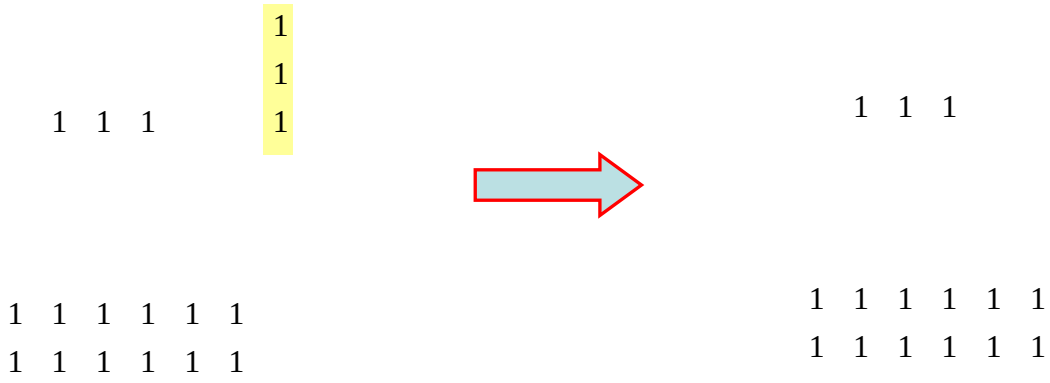
1 1 1 1 1 1

1 1 1 1 1 1

Aufgabe 7

X

$X \circ B_1$



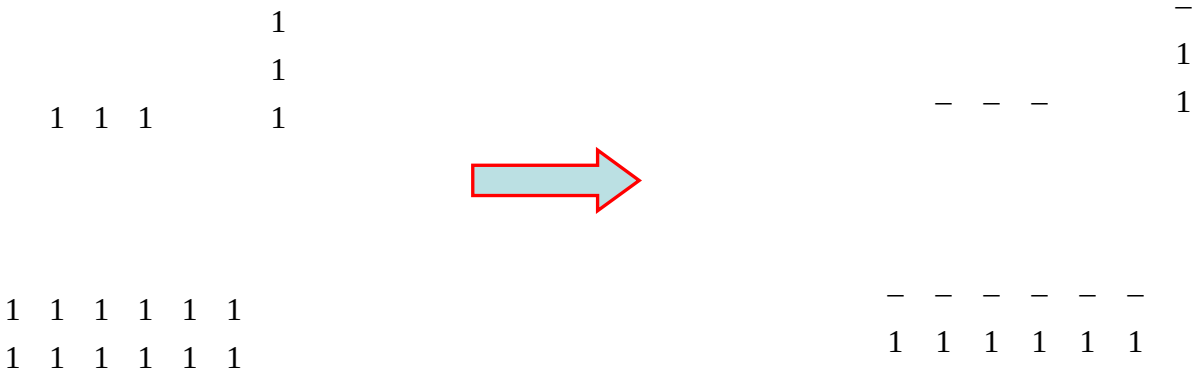
Aufgabe 7

B_2

1
1 ← (0,0)

X

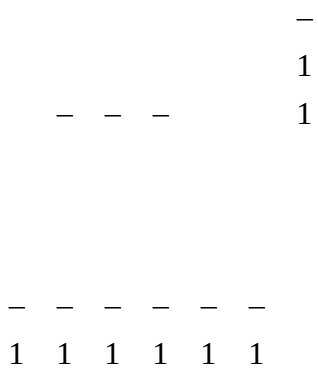
$X \ominus B_2$



Aufgabe 7

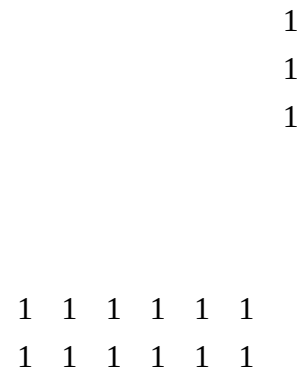
B_2

$X \ominus B_2$



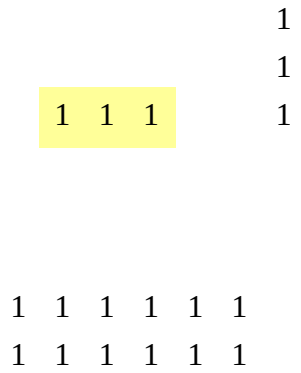
$\oplus B_2$

$X \circ B_2$

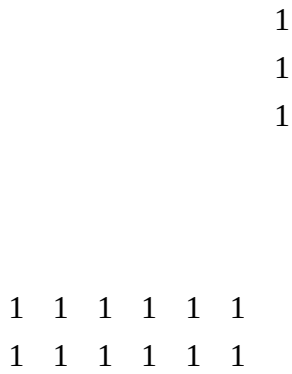


Aufgabe 7

X



$X \circ B_2$



Aufgabe 7

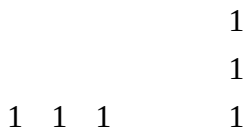
B_3

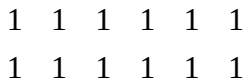
1 1
1 1

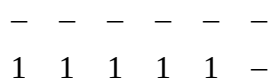
(0,0)

X

$X \ominus B_3$







Aufgabe 7

B_3

1 1
1 1

(0,0)

$X \ominus B_3$

$X \circ B_3$

– – – –
– – – – – –
1 1 1 1 1 –

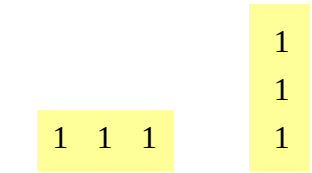
$\oplus B_3$

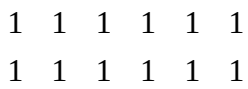
1 1 1 1 1 1
1 1 1 1 1 1

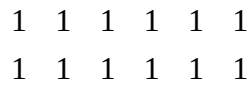
Aufgabe 7

X

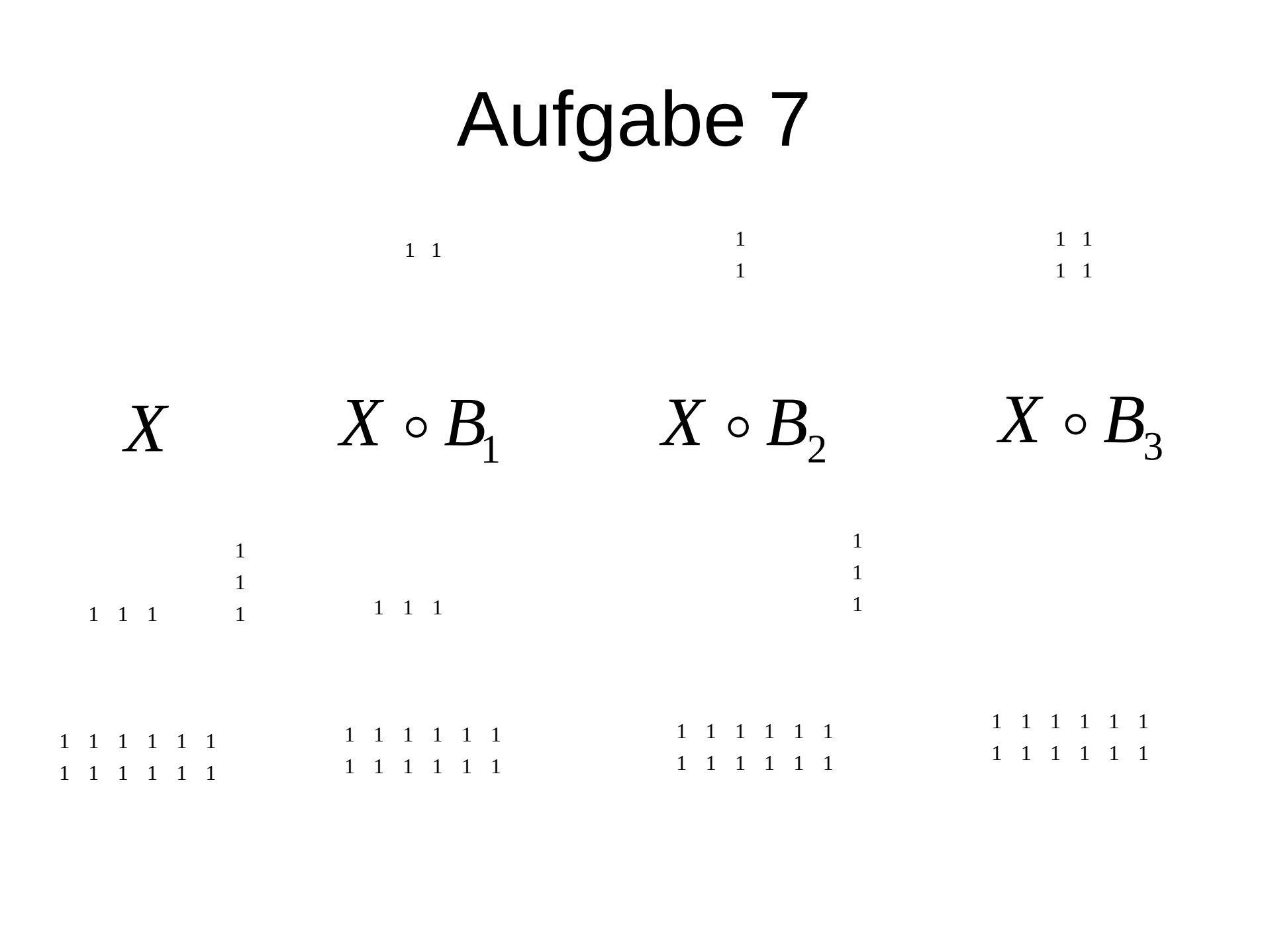
$X \circ B_3$







Aufgabe 7


1 1
1
1
1 1
1 1
X
$X \circ B_1$
$X \circ B_2$
$X \circ B_3$
1
1
1
1 1 1
1 1 1
1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1

Aufgabe 8

Beweisen Sie:

$$X \circ B = \bigcup_{y \in X \ominus B} B_y = \bigcup_{B_y \subseteq X} B_y$$

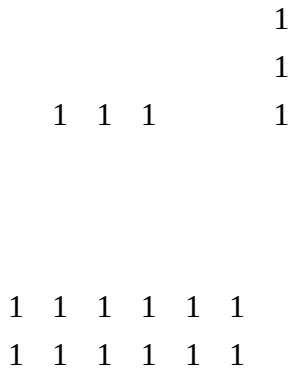
Aufgabe 8

$$X \oplus B = \bigcup_{b \in B} X_b = \bigcup_{x \in X} B_x$$

$$X \circ B = (X \ominus B) \oplus B = \bigcup_{y \in X \ominus B} B_y = \bigcup_{B_y \subseteq X} B_y$$

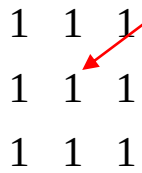
Aufgabe 9

X



B_3

B_4



Berechne: $X \bullet B_3$ $X \bullet B_4$

Vergleiche die Ergebnisse.

Aufgabe 9

B_3

1 1
1 1

(0,0)

X

1
1
1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1

$X \oplus B_3$

1 1
1 1
1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1 1

Aufgabe 9

B_3

1 1
1 1

(0,0)

$X \oplus B_3$

1 1
1 1
1 1 1 1 1 1
1 1 1 1 1 1

$\ominus B_3$

$X \bullet B_3 = X$

1
1
1 1 1 1

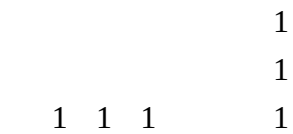
1 1 1 1 1 1
1 1 1 1 1 1

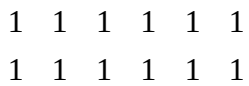
Aufgabe 9

B_4

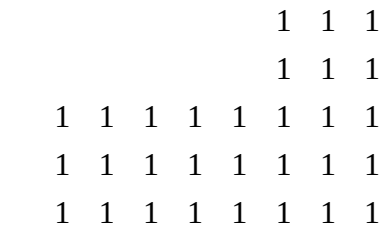
1 1 1
1 1 1
1 1 1

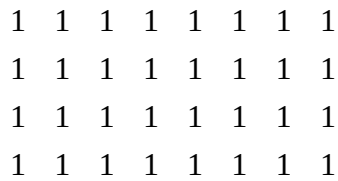
X





$X \oplus B_4$





Aufgabe 9

B_4

1 1 1
1 1 1
1 1 1

$X \oplus B_4$

1 1 1
1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1

1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1

$\ominus B_4$

$X \bullet B_4$

1
1
1 1 1 1 1 1

1 1 1 1 1 1
1 1 1 1 1 1

Aufgabe 10

X

1
1
1 1

$$B = (B_1, B_2)$$

$$B_1 = \{(0,0)\}$$

1 1 1
1 1 1

$$B_2 = \{(-1,0),(1,0),(0,-1),(0,1)\}$$

1

1

Berechne: $X \otimes B = (X \ominus B_1) \cap (X^C \ominus B_2)$

Alles oder Nichts Transformation

$$B_1, B_2, X \subseteq Z^n$$

$$B = (B_1, B_2) \quad B_1 \cap B_2 = \emptyset$$

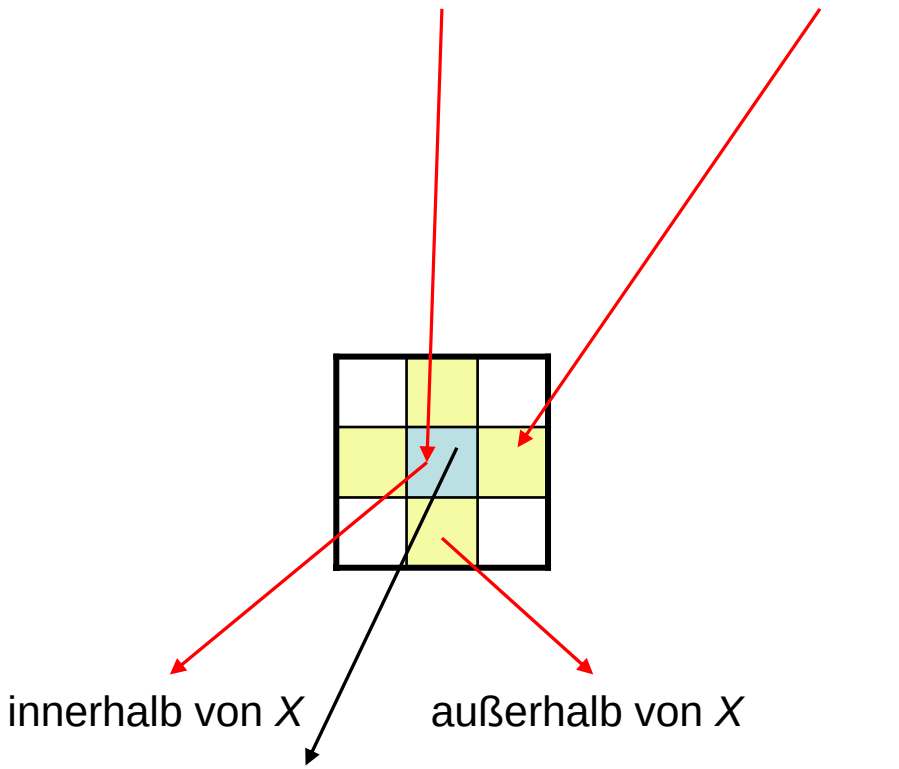
$$X \otimes B = (X \ominus B_1) \cap (X^C \ominus B_2)$$

Aufgabe 10

X

1
1
1 1
1 1 1
1 1 1
1
1

$$X \otimes B = (X \ominus B_1) \cap (X^C \ominus B_2)$$



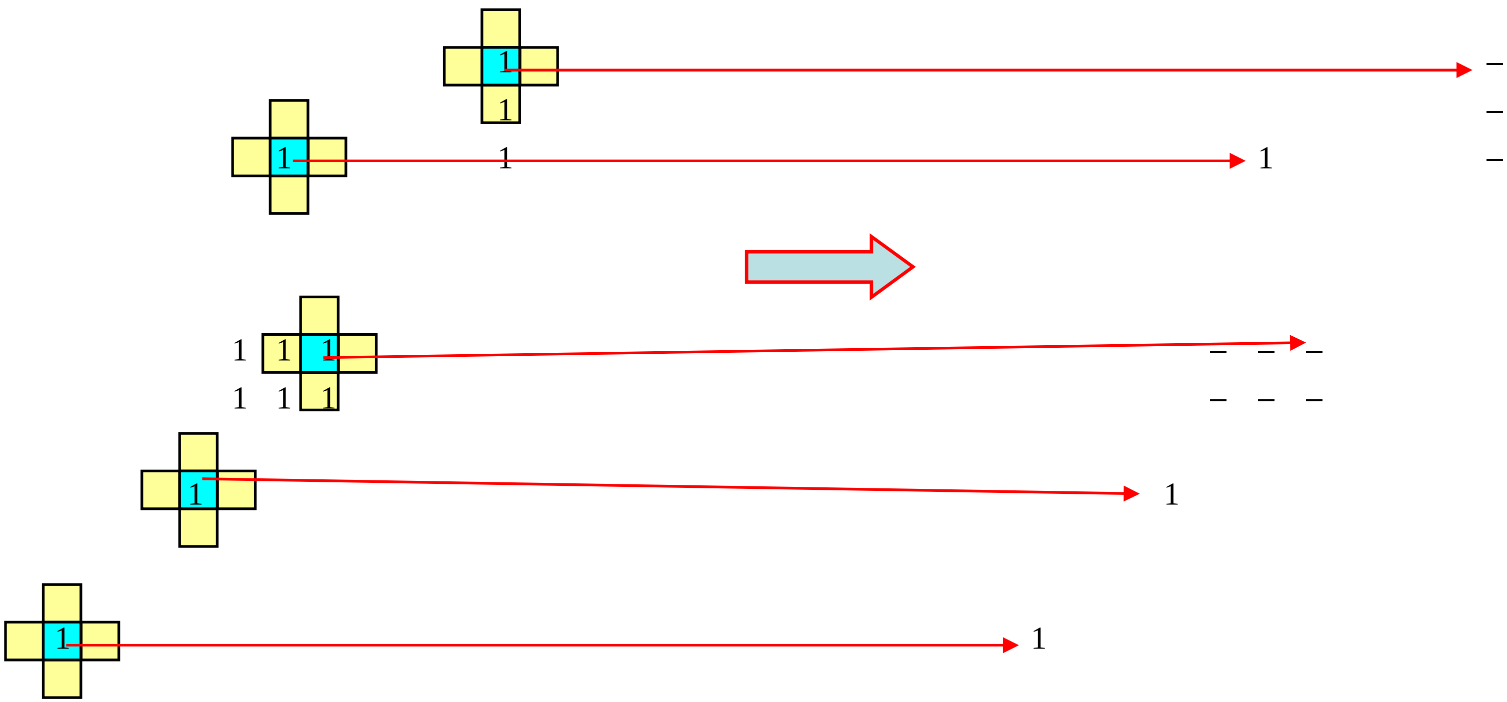
$B = (B_1, B_2)$

$B_1 = \{(0,0)\}$

$B_2 = \{(-1,0),(1,0),(0,-1),(0,1)\}$

Aufgabe 10

X $\qquad\qquad$ $X \otimes B$

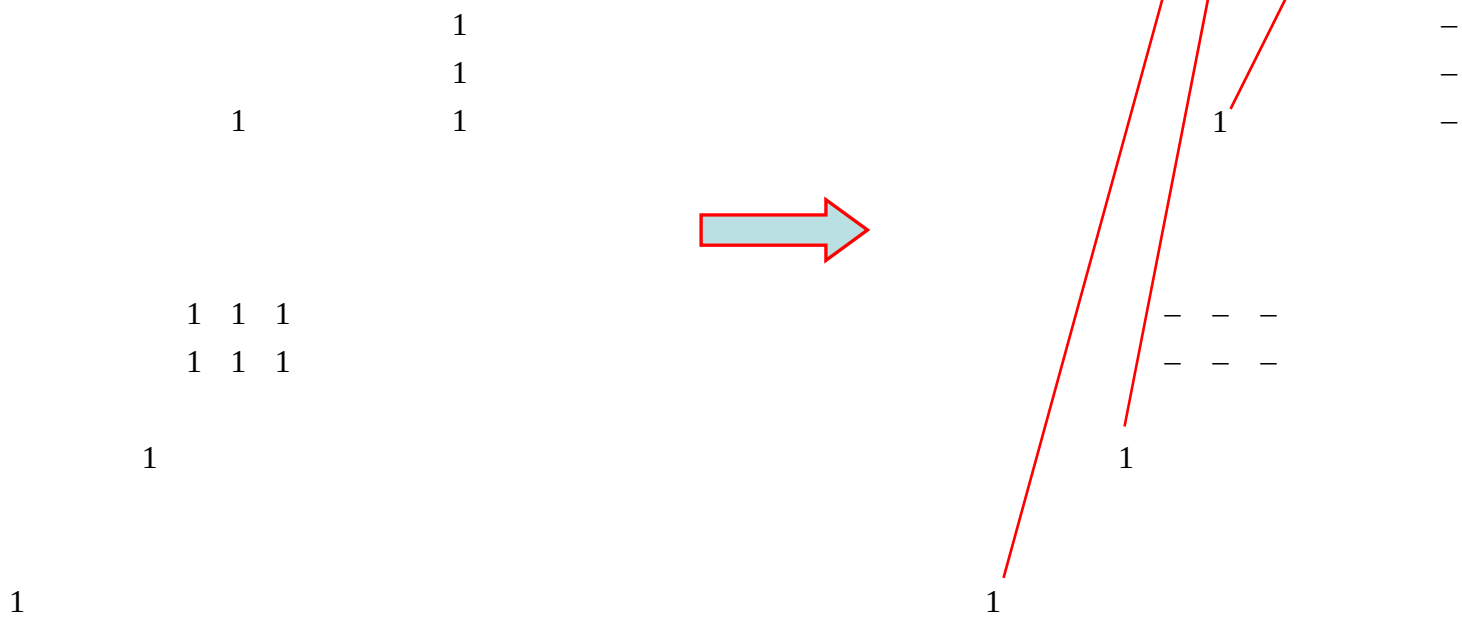


Aufgabe 10

X

$X \otimes B$

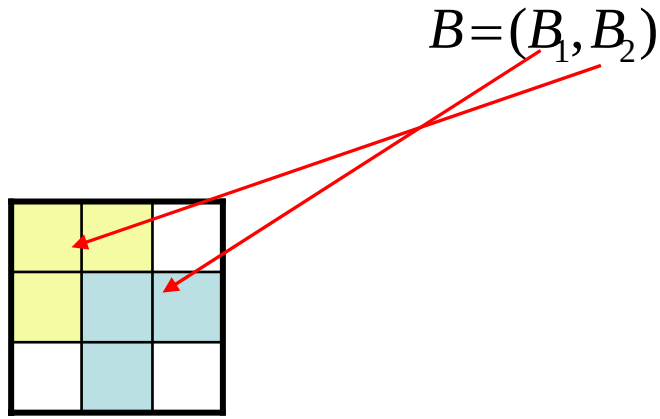
isolierte Punkte



Aufgabe 11

X

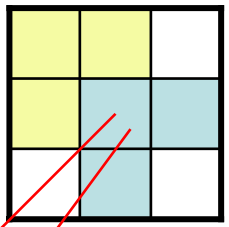
1	1	1
1	1	1
	1	
	1	
1	1	1
1	1	1



Berechne: $X \setminus X \otimes B$

Aufgabe 11

$B=(B_1,B_2)$



X

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ & 1 & \\ & 1 & \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

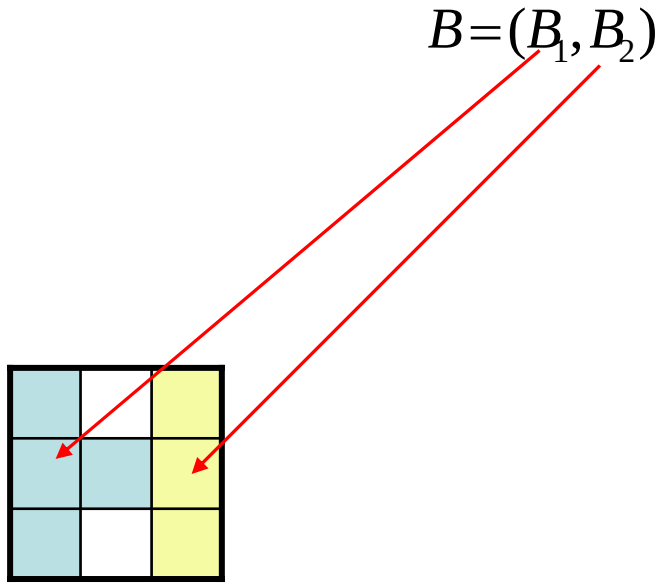
$X \setminus X \otimes B$

$$\begin{matrix} - & 1 & 1 \\ 1 & 1 & 1 \\ & 1 & \\ & 1 & \\ - & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

Aufgabe 12

X

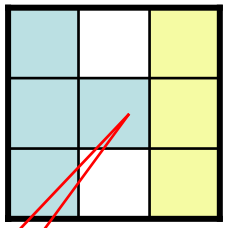
1 1 1
1 1 1
  1
  1
1 1 1
1 1 1



Berechne: $X \setminus X \otimes B$

Aufgabe 12

$B=(B_1,B_2)$



X

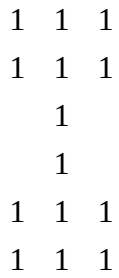
$X \setminus X \otimes B$

1	1	1
1	1	1
	1	
	1	
1	1	1
1	1	1

1	1	1
1	1	–
	1	
	1	
1	1	–
1	1	1

Aufgabe 13

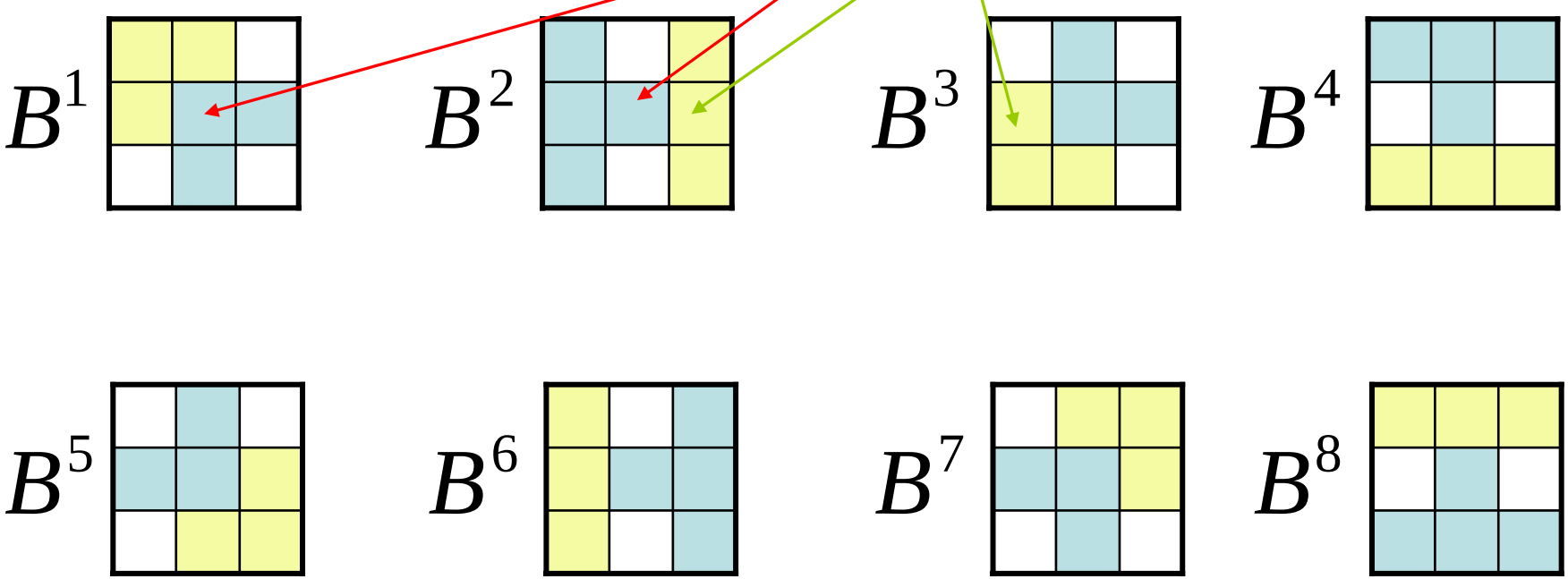
X



Wenden Sie den Skelettierungsalgorithmus aus der Vorlesung (3.4) an.
Betrachten Sie auch andere Reihenfolgen der Strukturelemente.

Skelettierung

$X \setminus X \otimes B^i$ $\quad B^i = (B_1^i, B_2^i)$ $\quad i = 1, \ldots 8$

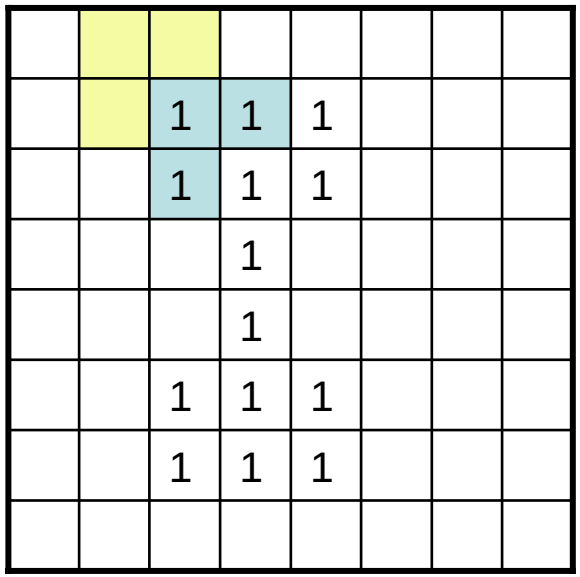


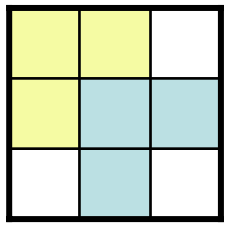
Aufgabe 13

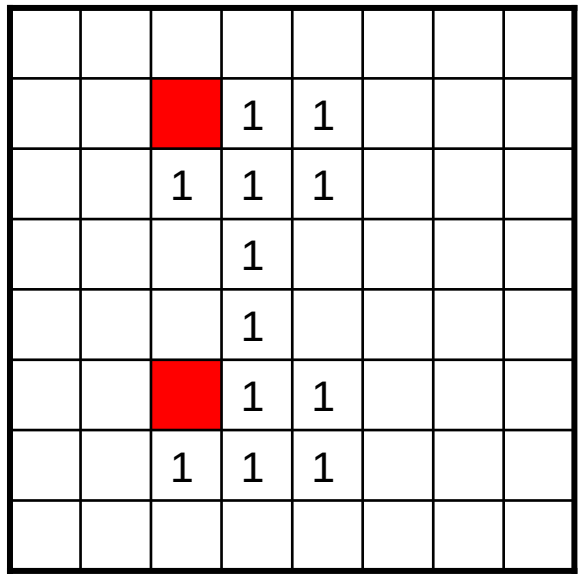
Anwendungsreihenfolge:

$$B^1, B^2, B^3, \ldots, B^8, B^1, \ldots, B^8, B^1, \ldots$$

$$X \setminus X \otimes B^1$$

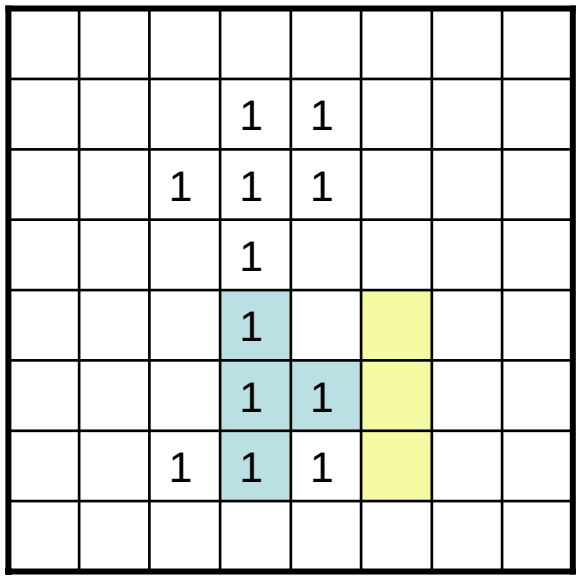


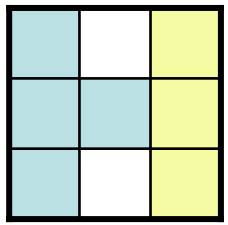


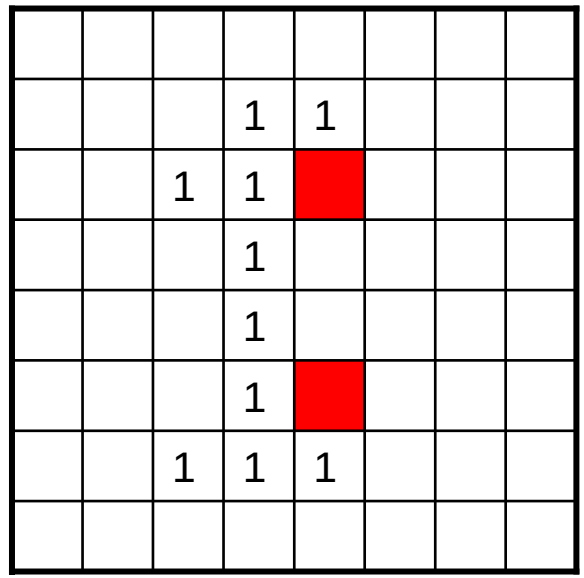


Aufgabe 13

$$X \setminus X \otimes B^2$$

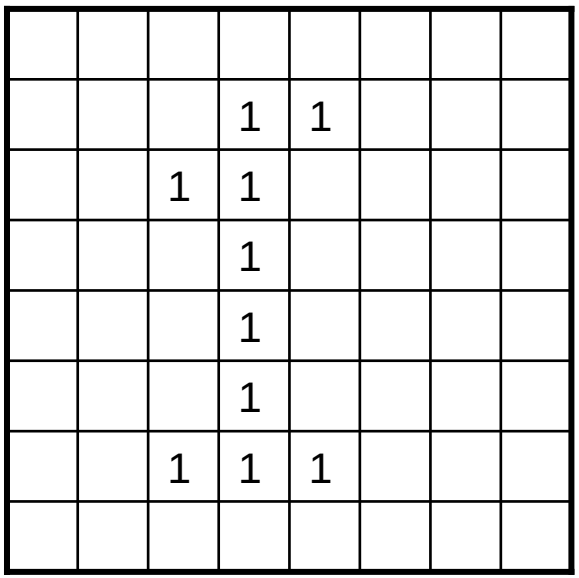


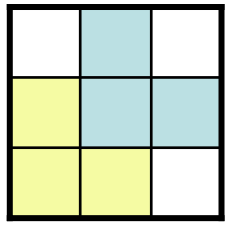


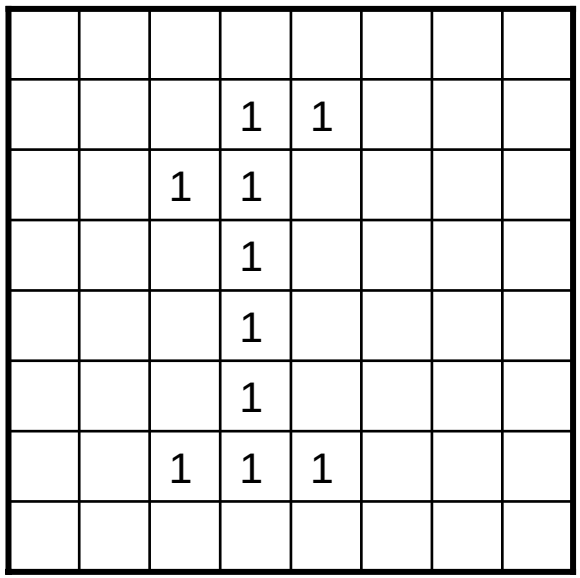


Aufgabe 13

$$X \setminus X \otimes B^3$$



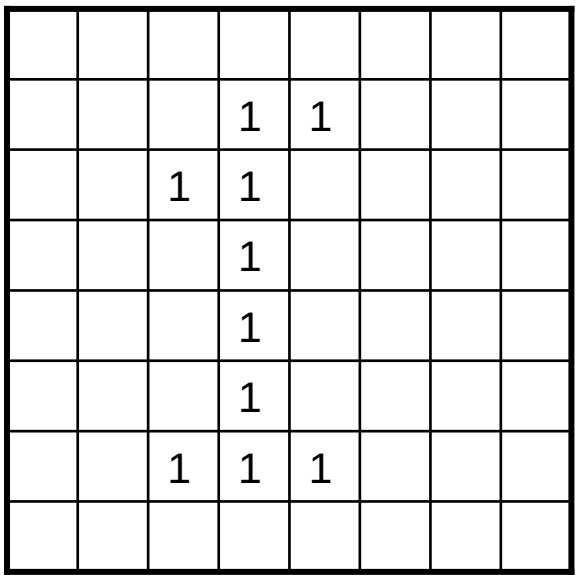


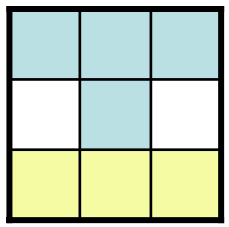


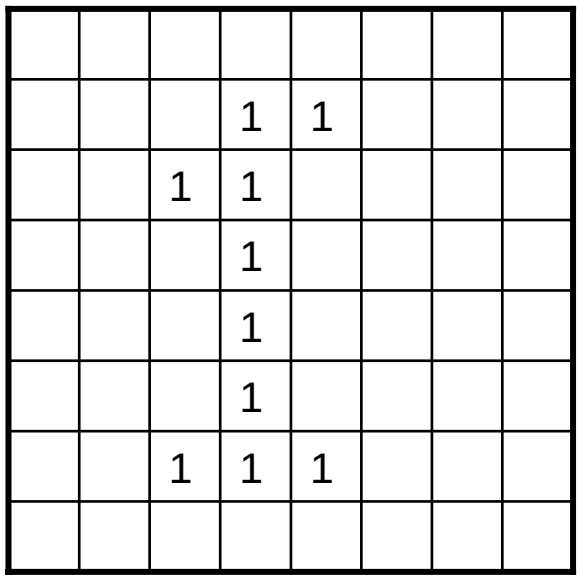
keine Änderung

Aufgabe 13

$$X \setminus X \otimes B^4$$





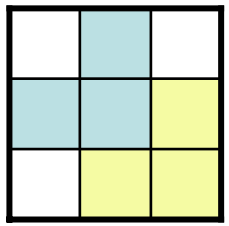


keine Änderung

Aufgabe 13

$$X \setminus X \otimes B^5$$

			1	1			
		1	1				
			1				
			1				
			1				
		1	1	1			

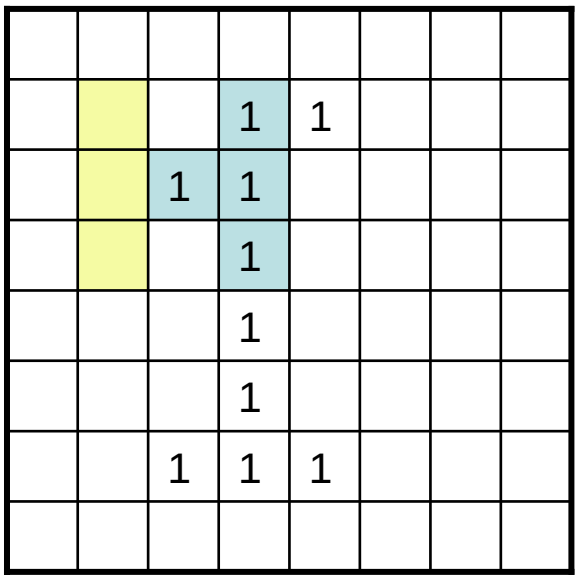


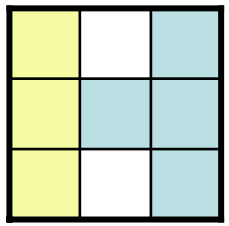
			1	1			
		1	1				
			1				
			1				
			1				
		1	1	1			

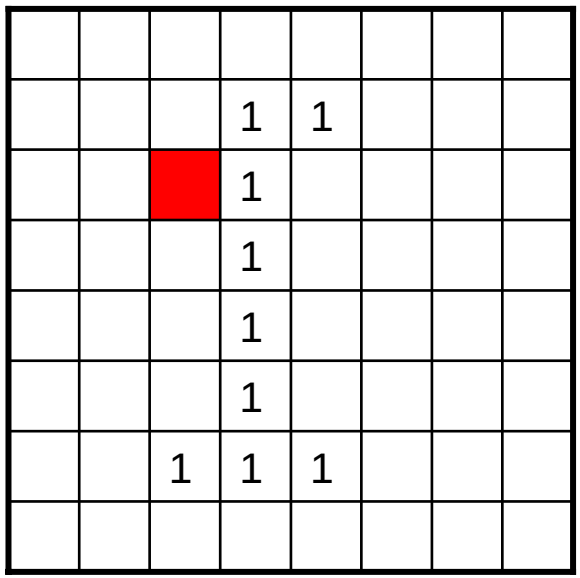
keine Änderung

Aufgabe 13

$$X \setminus X \otimes B^6$$

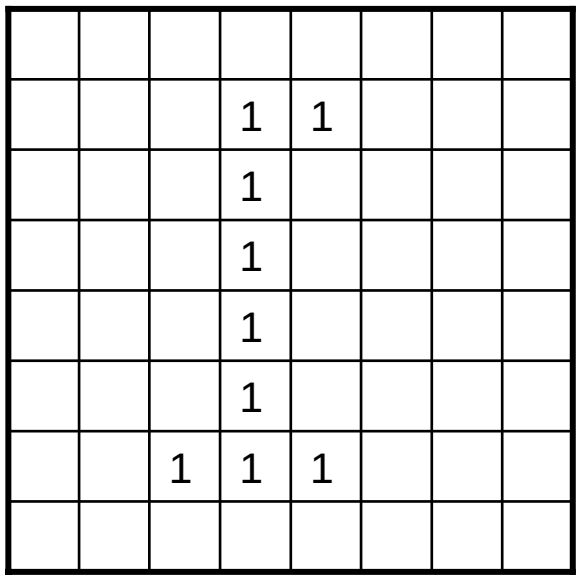


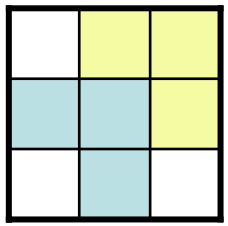


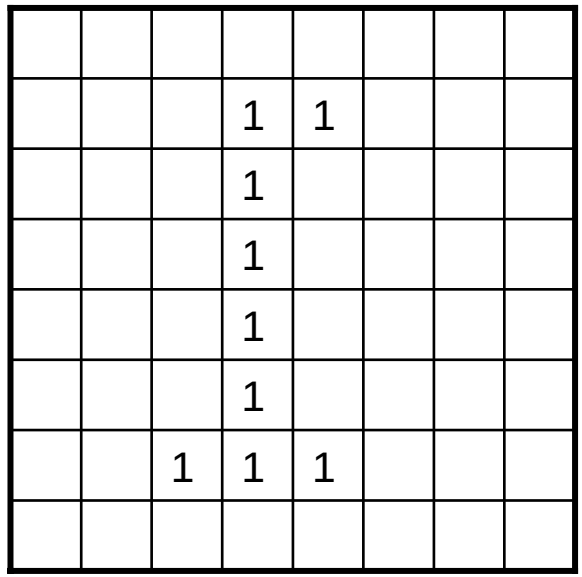


Aufgabe 13

$$X \setminus X \otimes B^7$$



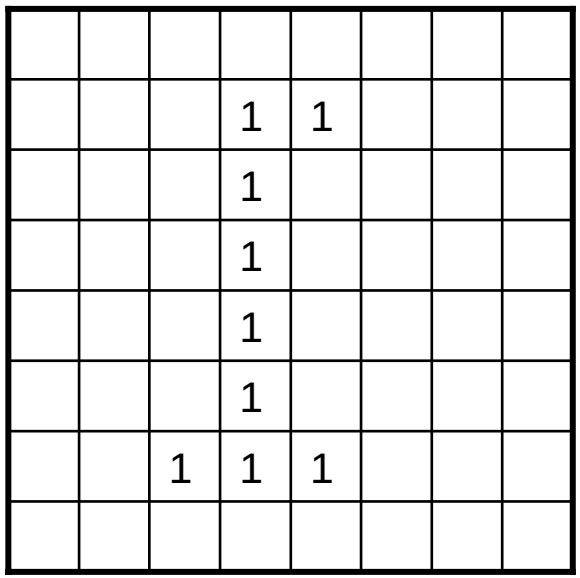


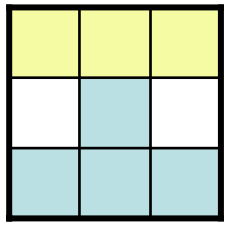


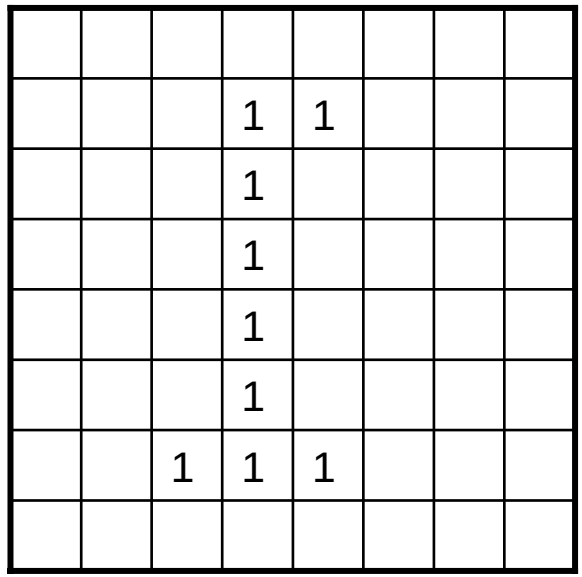
keine Änderung

Aufgabe 13

$$X \setminus X \otimes B^8$$





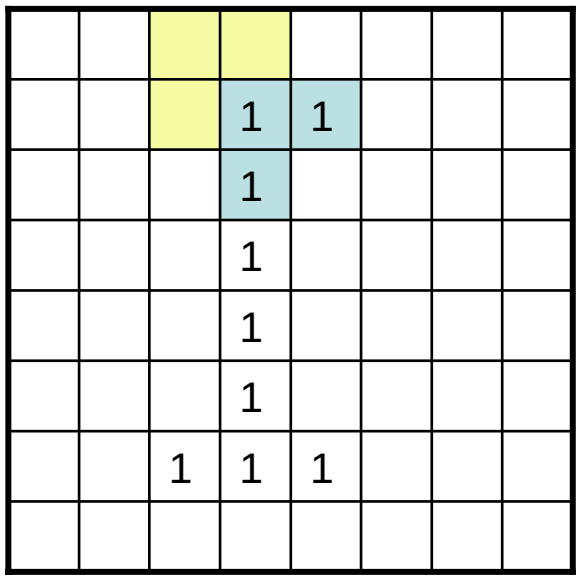


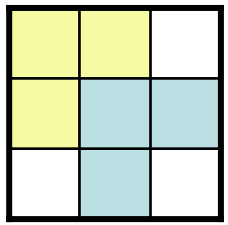
keine Änderung

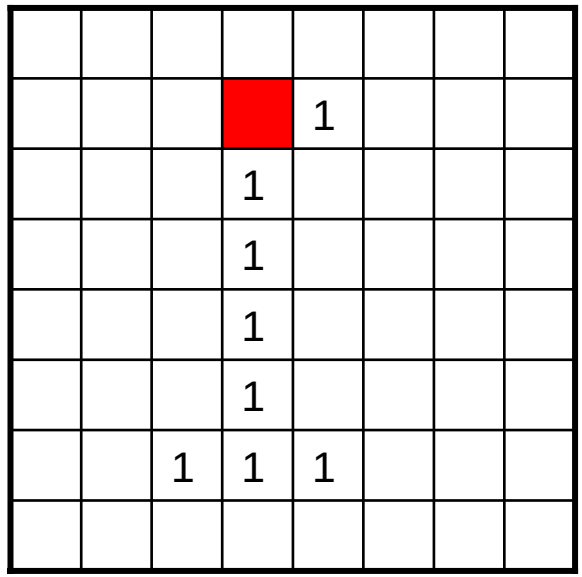
Aufgabe 13

$$X \setminus X \otimes B^1$$

wir beginnen wieder von vorn

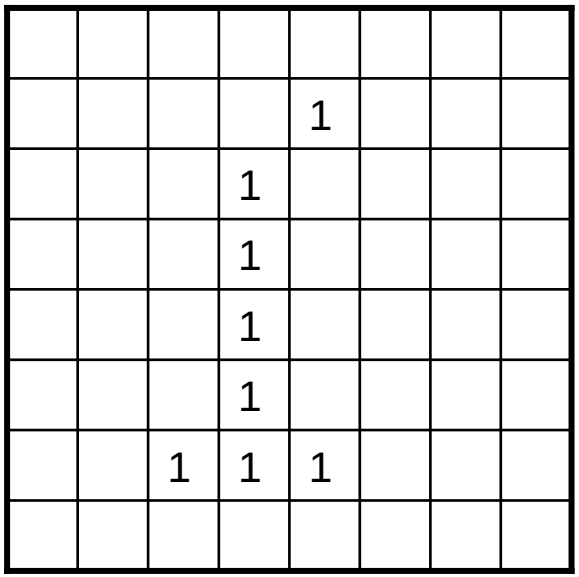


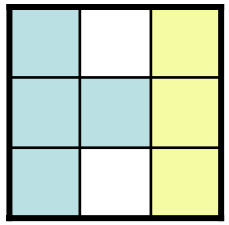


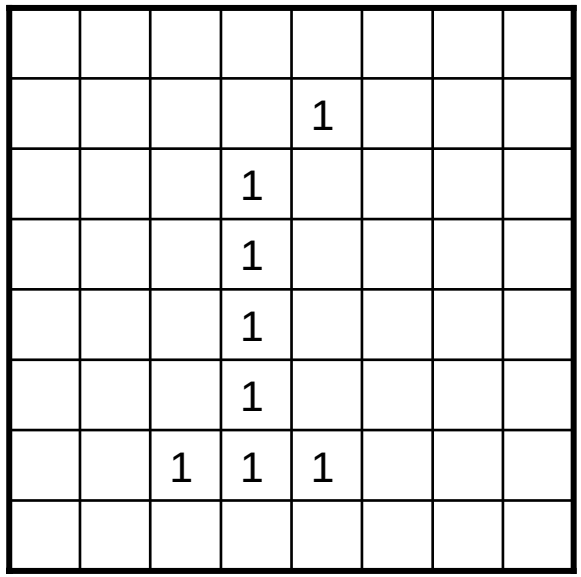


Aufgabe 13

$$X \setminus X \otimes B^2$$



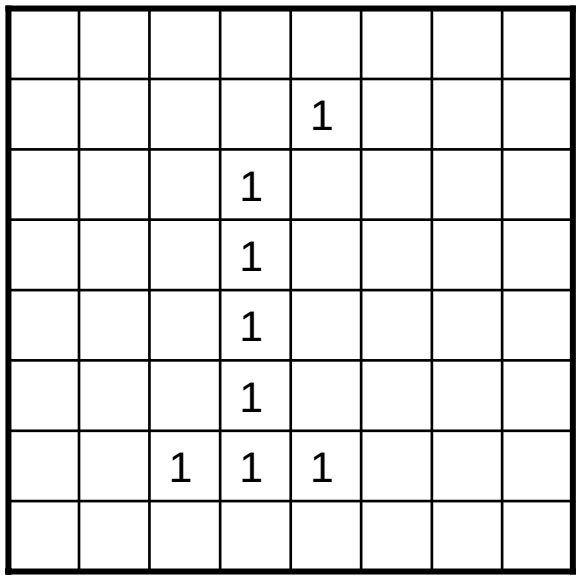




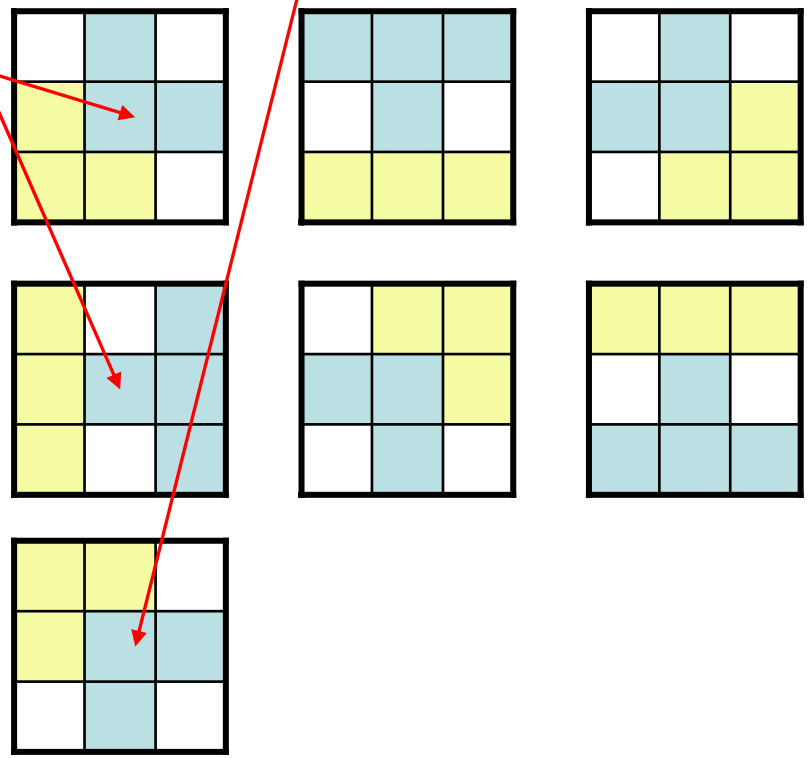
keine Änderung

Aufgabe 13

$$B^3, B^4, B^5, B^6, B^7, B^8, B^1$$



keine Änderung

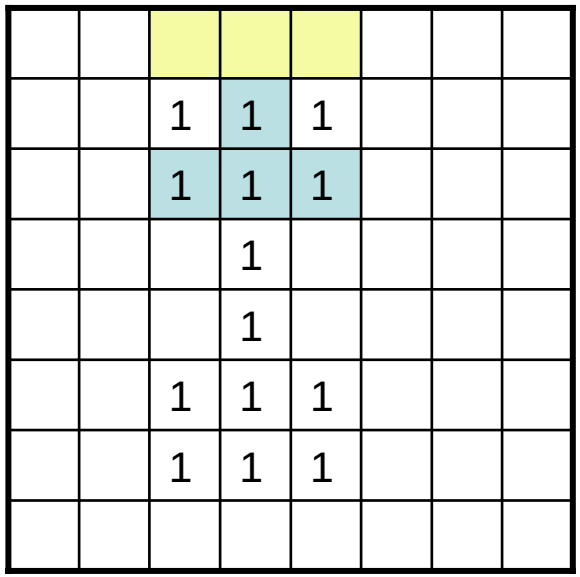


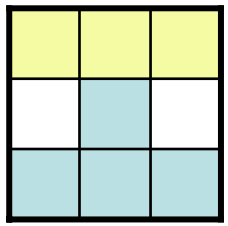
Aufgabe 13

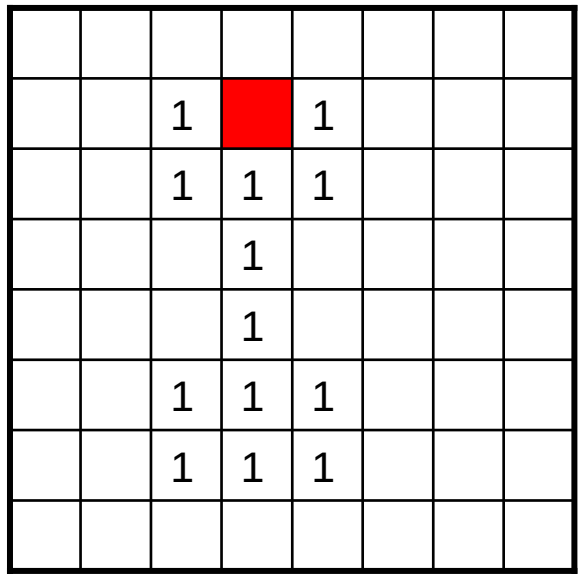
Anwendungsreihenfolge:

$$B^8, B^7, B^6, \ldots, B^1, B^8, \ldots, B^1, B^8, \ldots$$

$$X \setminus X \otimes B^8$$

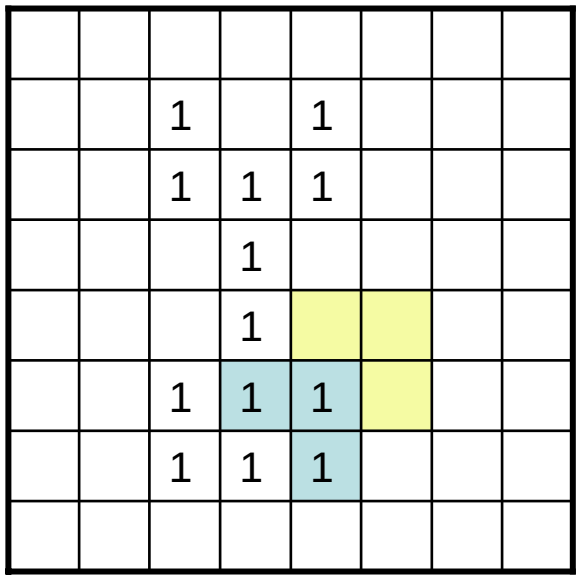


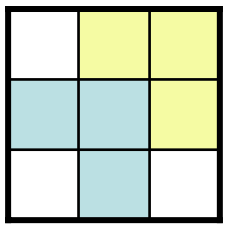


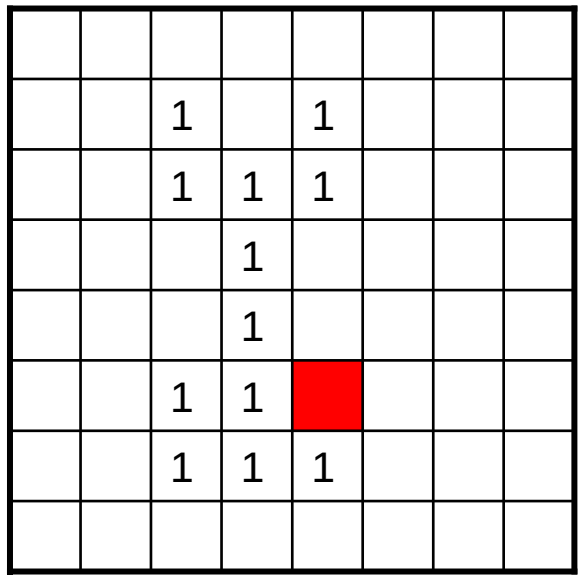


Aufgabe 13

$$X \setminus X \otimes B^7$$

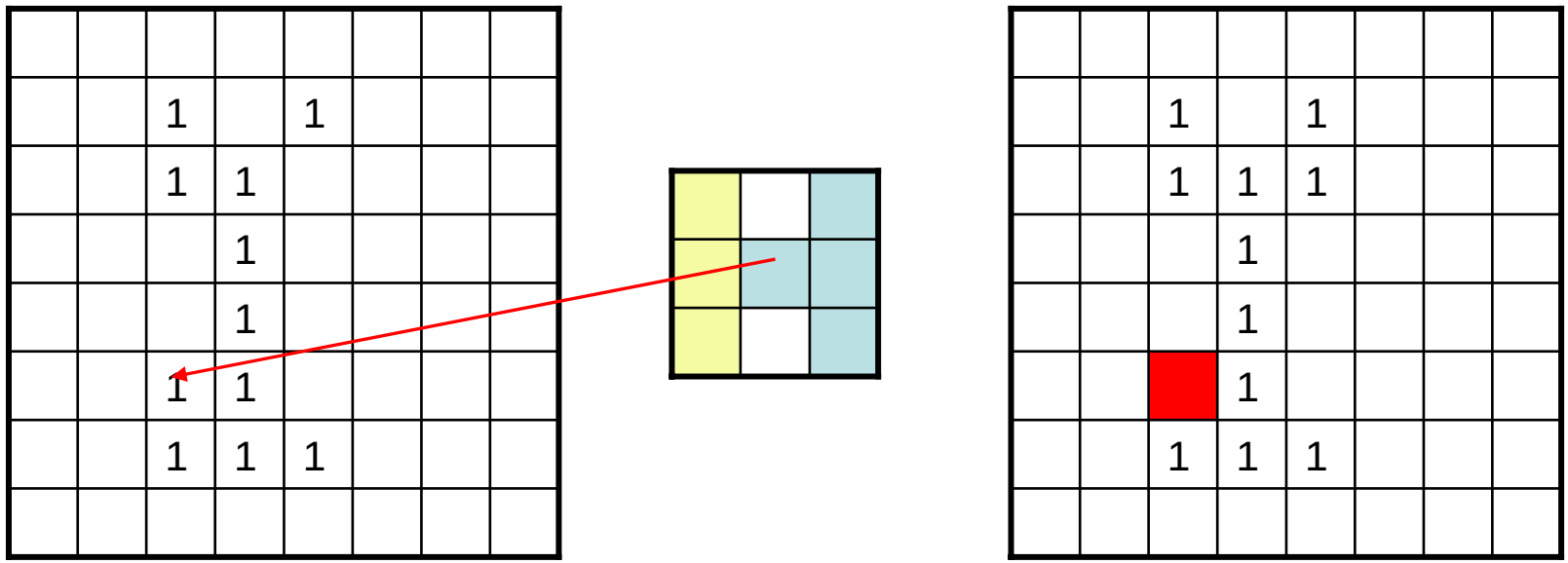






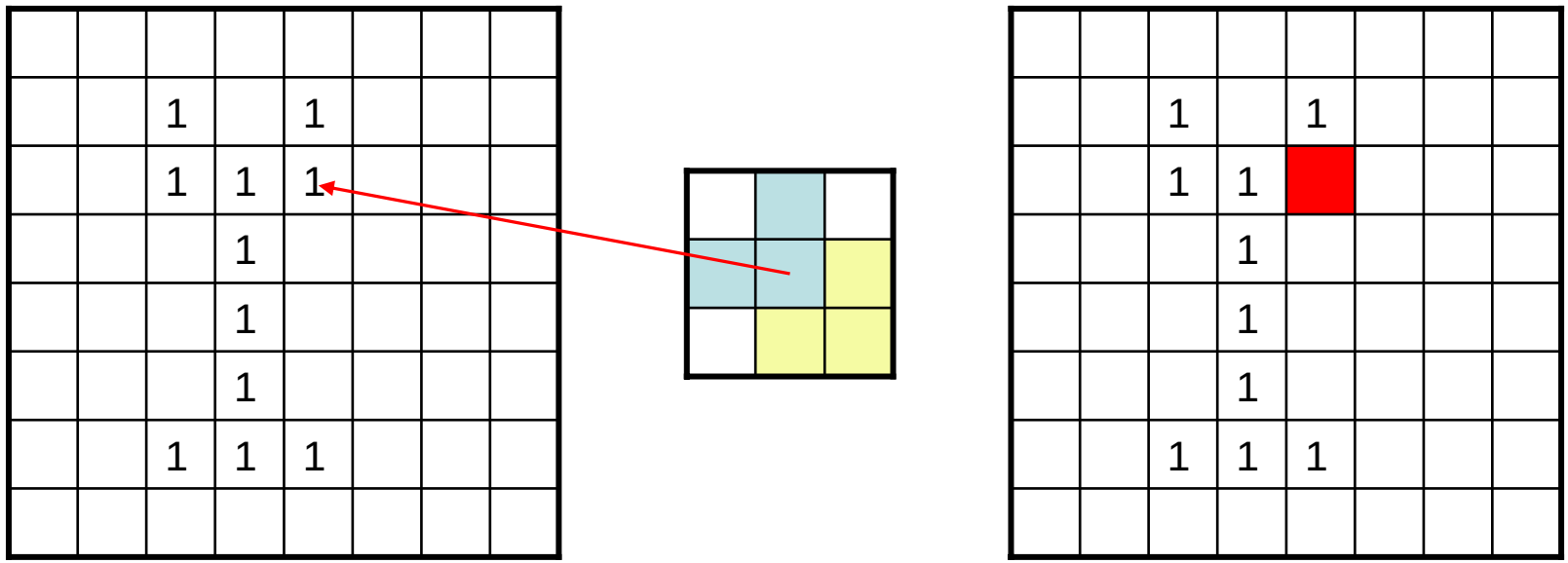
Aufgabe 13

$$X \setminus X \otimes B^6$$



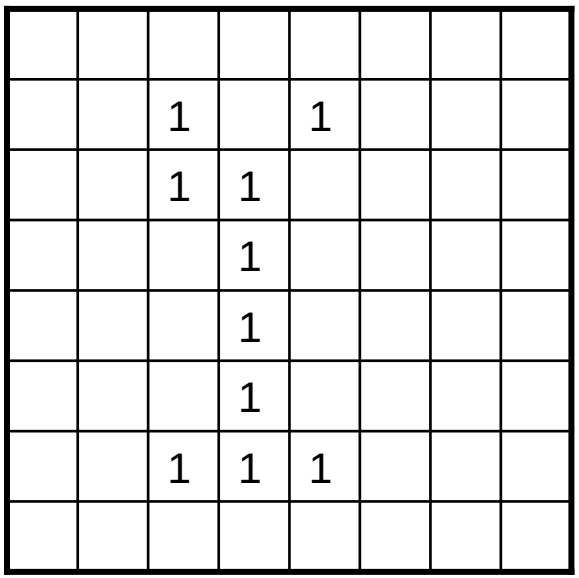
Aufgabe 13

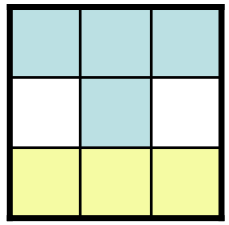
$$X \setminus X \otimes B^5$$

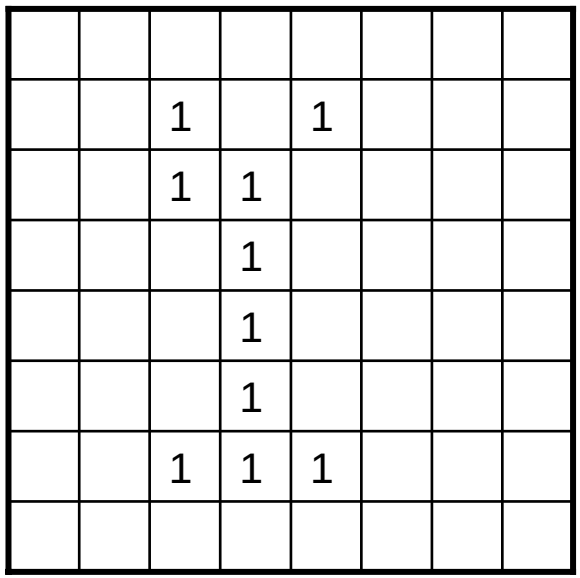


Aufgabe 13

$$X \setminus X \otimes B^4$$



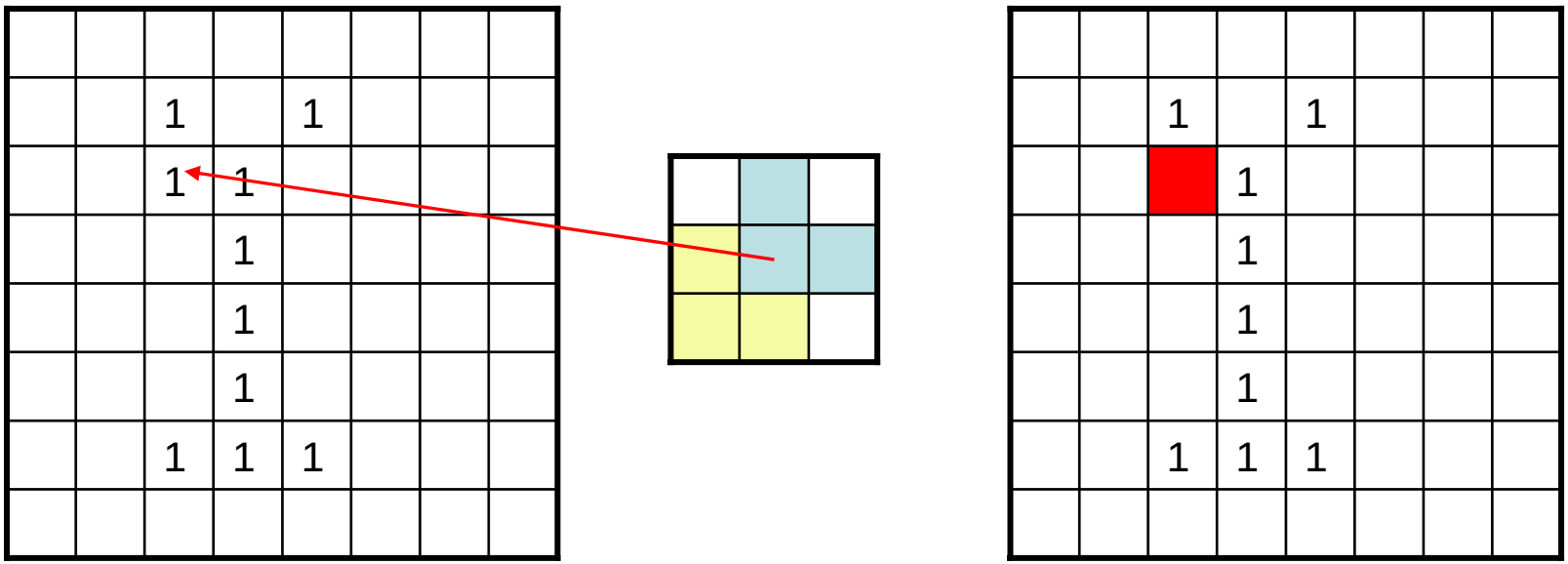




keine Änderung

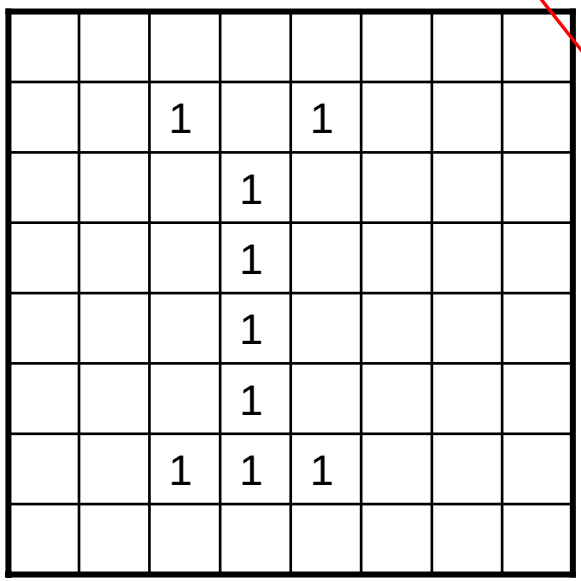
Aufgabe 13

$$X \setminus X \otimes B^3$$

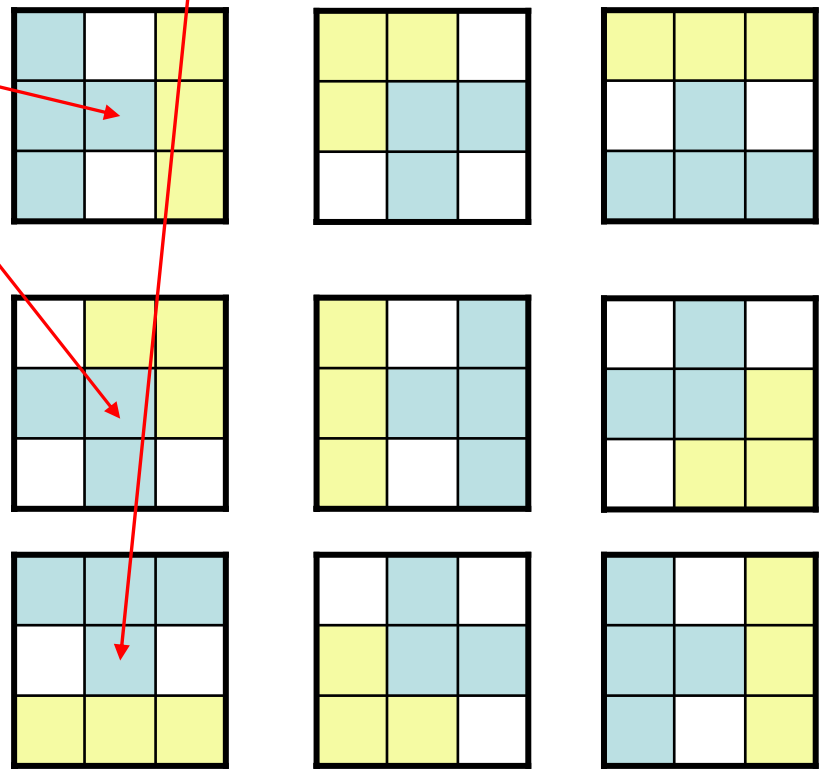


Aufgabe 13

$$B^2, B^1, B^8, B^7, B^6, B^5, B^4, B^3, B^2$$

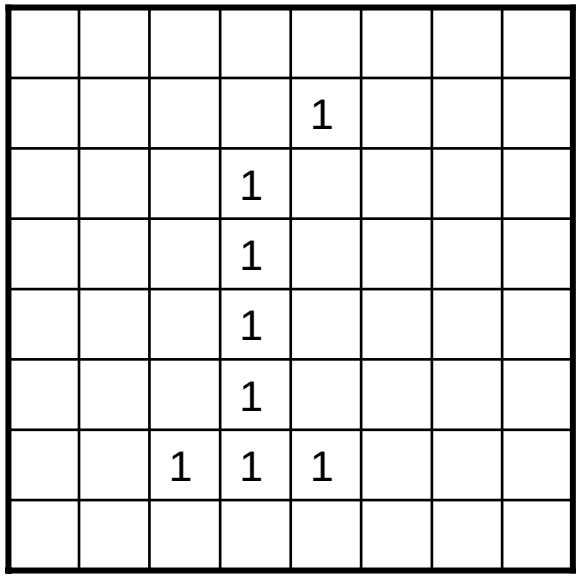


keine Änderung



Aufgabe 13 – Ergebnisse

$B^1, B^2, B^3, \ldots, B^8, B^1, \ldots, B^8, B^1, \ldots$



$B^8, B^7, B^6, \ldots, B^1, B^8, \ldots, B^1, B^8, \ldots$