Space-Efficient Traffic Protocols for Intelligent Crossroads

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***Abstract*— With the advent of autonomous driving, intelligent crossroads aim to substitute conventional traffic lights by interleaving vehicles crossing in all possible directions. To this end, there must be sufficiently large gaps between vehicles on the different lanes, which needs to be enforced by a *traffic protocol* taking vehicles' dimensions into account. In particular, existing such protocols are designed for the longest possible vehicle resulting in *space-hungry* intersections, that require modifications in the infrastructure (in particular, broader roads/lanes). Moreover, these do not allow for vehicles that are exceptionally longer than the ones considered at design time (e.g., extra long trucks, or buses, etc.). In this paper, to overcome this limitation, we present a traffic protocol which handles overlength vehicles as exceptions to compensate for their low probability of occurrence, relaxing space requirements on the intersections. We perform a detailed analysis and comparison showing that the proposed approach leads to high vehicle throughput while keeping intersections small, in particular, as overlength vehicles become longer and less frequent.**

I. Introduction

In future, autonomous driving will allow for a new set of applications that are not possible or at least difficult to implement with conventional vehicles. In this paper, we are concerned with one such application, viz., intelligent crossroads.

An intelligent crossroad attempts to regulate traffic flow with the aim of reducing congestion and preventing traffic jams. The idea is to replace traditional traffic lights by a roadside unit (RSU) that synchronizes approaching vehicles according to a predefined traffic protocol or policy.

The traffic protocol synchronizes vehicles on the different lanes such that they can cross the intersection in all possible directions without needing to stop. To this end, a safe inter-vehicle separation needs to be guaranteed on each lane, which depends on vehicles' dimensions, in particular, on the lengths of vehicles on the conflicting lanes. For example, the separation between two vehicles traveling from north to south should be sufficiently large for a vehicle to cross from east to west in between them.

Existing protocols, such as the Ballroom Intersection Protocol (BRIP)[1], account for this by considering the worst case, i.e., assuming that the largest possible vehicles are always present at the intersection. This leads to significant space requirements, which either forces the intersection to be built from scratch or imposes severe restrictions on the maximum length of a vehicle allowed at the crossroad.

In this paper, to overcome this predicament, we propose a traffic protocol called to as LTR (Left turn, Through, Right turn) that handles overlength vehicles as exceptions, thereby, relaxing space requirements on the intersection. Similar to BRIP, the proposed LTR divides the intersection into equal-sized, square sectors. However, in contrast to BRIP, LTR does not require a vehicle to fully fit into one such sector, but the size of sectors is rather chosen independently. Since an overlength vehicle occupies more sectors than others, they introduce an overlength penalty, i.e., a temporary increase in the inter-vehicle separation on conflicting lanes.

A detailed analysis and comparison shows that the proposed LTR allows for a considerably more space-efficient design of intelligent crossroads than BRIP. In particular, for overlength vehicles of up to 15m, LTR reduces the intersection size to 1/4 of that of BRIP. In addition, the LTR outperforms BRIP in terms of vehicle throughput for overlength vehicles of 15m onwards. For example, when the side/length of a sector is set to $5m$ and overlength vehicles are as long as 30m, i.e., the length of a tram, LTR results in 50 more vehicles per minute as discussed later in detail.

Structure of the paper: In Section II, we discuss related work, while Section III outlines the basic assumptions and definitions used later for the proposed traffic protocol LTR, which is described in Section IV. Section V presents BRIP as the main object of comparison. With both protocols introduced, Section VI presents an analysis and comparison of the performance with respect to throughput and space requirements on the intersection. Finally, Section VII concludes the paper.

II. Related Work

Since this is a relatively new area, only a few works investigate design and analysis methods for intelligent crossroads.

The general idea of scheduling cars at a road intersection was first proposed in [2][3]. Here, vehicles are coordinated by a traffic management system based on reservations. This leads to a synchronized crossing pattern and constitutes the basis of the proposed traffic protocol in this paper. In our case, the crossroad is assumed to be completely filled with vehicles according to the reservation logic which fixes the inter-vehicle spacing.

Further works concerning collision avoidance using traffic protocols and vehicular networks are [4][5]. These make use of the idea of cooperative maneuvers in the context of autonomous driving. Collisions are avoided through Vehicle-to-Vehicle (V2V) communication and a more complex traffic management system than the ones mentioned before. The

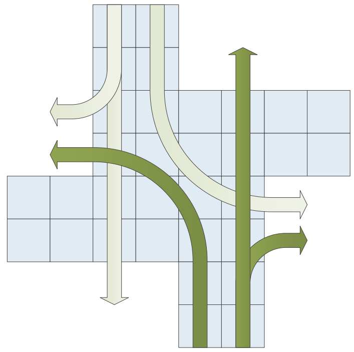

Fig. 1: Basic crossroad layout of the proposed protocol: with one left-turn and one shared through/right-turn lane. The underlying squares highlight the concept of sectors used throughout this paper.

goal of these works is to avoid collisions and, in contrast to the proposed approach, not to optimize throughput and space requirements on the intersection.

This concept then evolved into the already mentioned *Ballroom Intersection Protocol* (BRIP) [1], which is designed to increase throughput at the intersection. BRIP considers several crossroad layouts (i.e., with different combinations of through and turn lanes) and proposes a tightly planned schedule of vehicles. To this end, the intersection is divided into square *sectors* — one sector per lane — with sides S equal to at least $L + W$, where L and W are the length and width of the longest possible vehicle that is allowed to cross the intersection. However, this imposes non-negligible limitations. For example, trucks with 10m length and 3m width lead to intersections of up to $52m$ for a double-lane setup, which is more than twice the size of a normal double-lane intersection. As a consequence, BRIP is not suitable for existing infrastructure and, in the end, requires the intersection to be built from scratch.

In [6], autonomous vehicles cross an unsignalized intersection through collaborative collision avoidance, presented as an optimization problem based on vehicles' speed ratios. In contrast to this paper, the dimensions of vehicles and of the intersection itself are not discussed, since the focus is on conflict resolution rather than throughput and space efficiency.

A complete safety verification for collision avoidance at road intersections is presented in [7], which similarly to this paper also accounts for vehicles' lengths. However, in contrast to the proposed approach, the space requirements on the intersection and throughput of vehicles are not discussed.

The proposed approach of this paper synchronizes vehicles at an intersection with the aim of maximizing throughput and, hence, is rather in line with BRIP than with works in [6][7]. However, as discussed later, our approach makes better use of the physical space and does not require changing existing infrastructure

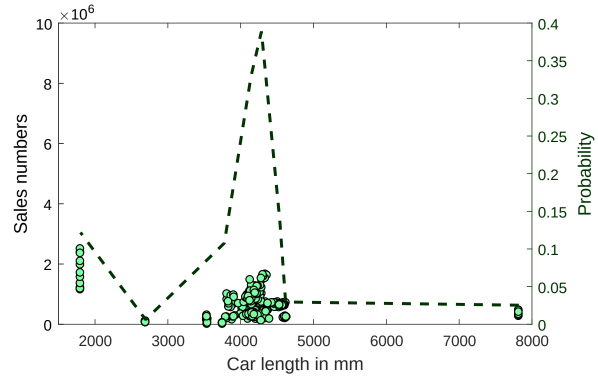

Fig. 2: Vehicle length data and corresponding length distribution from [8].

III. Assumptions and Definitions

In this paper, we assume *right-hand traffic* for all figures and explanations. As already mentioned, the intersection is divided into same-size, square sectors. For simplicity, the side or *length* of a sector S is chosen to be equal to the width of a lane — see Fig. 1.[1] We further consider that $S = k \times \sigma$ holds for any positive integer number k, where σ is defined as an *atomic fraction* of S. In this paper, for simplicity, we fix $\sigma = 1m$, however, it can be chosen arbitrarily as long as it allows fulfilling the above condition on S.

Further, we define *vehicle period* as the distance from front bumper to front bumper of two consecutive vehicles on the same lane. The vehicle period minus $\lceil \frac{L}{\sigma} \rceil$ — where L is the length of the leading vehicle — results in the inter-vehicle separation between these vehicles. We express vehicle period and inter-vehicle separation in terms of multiples of σ.

We assume that the RSU enforces the same constant speed for all vehicles entering the crossroad's *region of influence* given by a radius from the center of the intersection, e.g., 200m.[2] Further, we refer by *cycle* to the time required to cover a distance equal to S at that speed. Note that it takes a vehicle $1/k$-th of a cycle to cover a distance equal to σ.

Vehicles can have very different lengths with regular vehicles being around $5m$ long [11]. However, there are also exceptionally short cars [10] and motorbikes [9] that are shorter. In addition, trucks have lengths around $10m$ [11]. Based on sales statistics, it is possible to derive a probability distribution depending on vehicles' lengths as depicted in Fig 2.

For a given value of S, a vehicle either fits entirely in a sector or it requires more than one sector. If a vehicle requires more than one sector, we refer to it as an *overlength vehicle*. The amount of additional space it requires is referred to as *overlength penalty*, which can be computed as follow:

$$O = \max\left(0, \left\lceil \frac{L - S}{\sigma} \right\rceil\right), \qquad (1)$$

[1] In principle, it is possible to chose an arbitrary value for S, however, this might complicate the resulting protocols unnecessarily.

[2] Otherwise, the separation between vehicles will have to compensate for differences in speed complicating the overall design.

where L is again the length of the vehicle. This equation results in either a positive multiple of σ or zero.

One can increase S to reduce the overlength penalty, in particular, $O = 0$, if $S \geq L$ holds. This simplifies the traffic protocol, since vehicles travel one sector in one cycle. However, it leads to space-hungry intersections as already discussed. In this paper, since long vehicles are less probable as shown in Fig. 2, we opt to incorporate overlength penalty into the traffic protocol as we discuss later in more detail.

IV. Proposed Protocol LTR

In this section, we introduce our proposed traffic protocol for intelligent crossroads called LTR (Left turn, Through, Right turn) as shown in Fig. 1.

Again, the goal is to synchronize vehicles at the intersection such that they can safely cross in all possible directions without needing to stop. In this context, we identify two important metrics for quality: i) throughput, i.e., vehicles per time unit and ii) size of the intersection. While the size of the intersection depends on S and the number of lanes, throughput is affected by the vehicle period/inter-vehicle separation and, hence, by the pattern with which vehicles cross in the different directions.

We first consider the case of vehicles that fit entirely in one sector and then extend our analysis to overlength vehicles. Note further that the basic 2-lane layout of LTR can be extended to more complex intersections with multiple lanes in every direction.

Drive through only/drive through and right turns (Fig. 3): With respect to driving through in all directions, a synchronized crossing, where vehicles arrive at the intersection's borders simultaneously is as depicted in Fig. 3. To avoid collisions, this maneuver is divided into 6 different cycles — recall that vehicles cover a distance equal to S in one cycle. The resulting vehicle period, i.e., the distance from front bumper to front bumper, becomes $6S$ on each of the lanes. A shorter vehicle period may cause accidents between vehicles crossing in perpendicular directions. A longer such period reduces throughput.

Provided that vehicles have a constant speed, we can compute throughput by the ratio between vehicles over cycles, independent of the concrete speed value. That is, 4 vehicles cross the intersection in 6 cycles leading to $\frac{4}{6}$ vehicles per cycle.

Note that a vehicle turning right leaves the intersection within 3 cycles — see again Figs. 1 and 3. As a result, this does not affect the vehicle period when combined with driving through, which remains at $6S$. The vehicle period reduces to $3S$, only when all vehicles turn right at the same time. In this special case, the throughput becomes $\frac{4}{3}$ vehicles per cycle.

Drive through and left turns (Fig. 4): We know from above that 6 cycles are needed to allow 4 vehicles to drive through in all directions. After drive-through vehicles leave the intersection, a left-turn maneuver is started as depicted

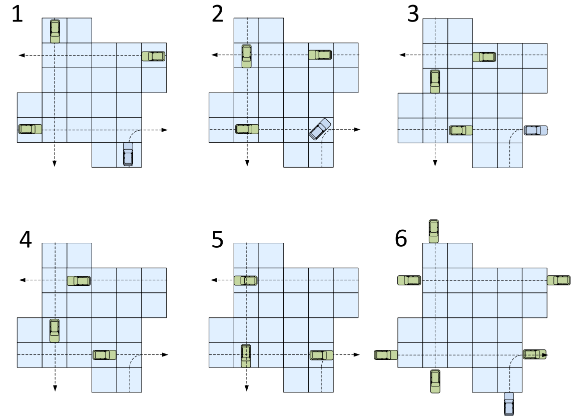

Fig. 3: Driving through in all four directions. The synchronized crossing maneuver is divided in 6 cycles and, hence, the vehicle period on each lane is equal to $6S$, where S is again the length of a sector.

in Fig. 4. When turning (left), a vehicle describes a curved trajectory t, shown in Fig. 5. Thereby, it covers a distance equal to a quarter of a circumference with radius $2.5S$ (under the assumption that vehicles go from the middle of one lane to the middle of the other). This can be calculated as follows:

$$t = \frac{\pi}{4} \cdot (2 \cdot 2.5S) = \frac{5}{4} \cdot \pi S = 3.9270S. \quad (2)$$

Note that this desynchronizes the left-turn trajectory by an amount equal to $(4 - \frac{5}{4} \cdot \pi) = 0.073$ cycles, i.e., it requires a non-integer number of cycles. In other words, left-turning vehicles exit the intersection 0.073 cycles before the 4-th cycle ends. This does not affect safety, since the vehicles are

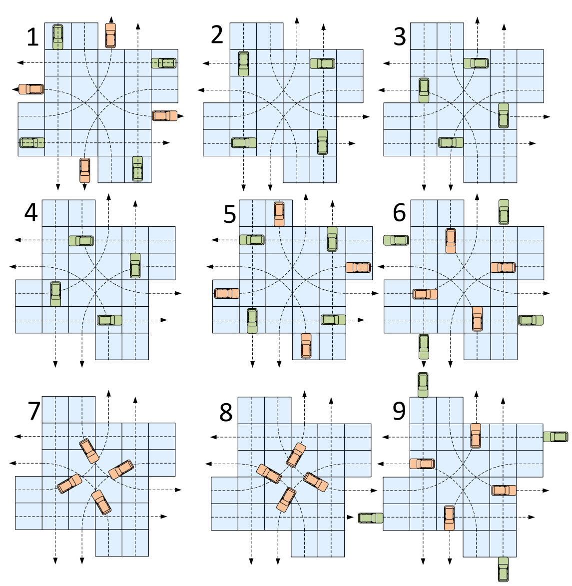

Fig. 4: Combined driving through and left turns in all four directions. Note that a throughput of 8 to 6 vehicles every 9 cycles is possible, depending on whether a *four-way synchronization* or a *two-way synchronization*, where only vehicles on opposite lanes cross simultaneously, can be implemented. This figure illustrates a four-way synchronization.

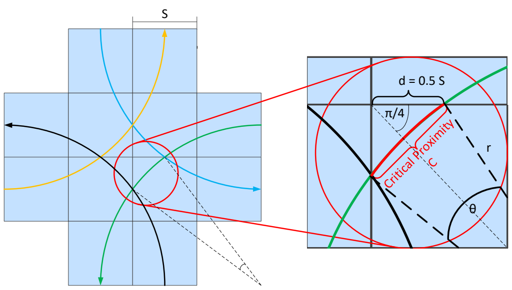

Fig. 5: Proposed four-way synchronization for left turns. If one vehicle exceeds a given maximum length, a two-way synchronization where only opposing lanes cross simultaneously must be enforced instead. This is due to the reduced distance between vehicles on perpendicular left-turn lanes.

no longer in the center of the intersection, and there still is sufficient space ($> 1.9S$) to a potential previous vehicle in the through direction.

In the best case, vehicles traverse the intersection in all possible four directions. We refer to this as a *four-way synchronization*, which is depicted in Fig. 4 and Fig. 5. However, due to conflicting trajectories, the distance between two vehicles turning left on perpendicular lanes is reduced to the following:

$$C = r \cdot \theta = 2.5S \cdot \theta = 0.7095S,$$

where θ is obtained using the Law of Sines — see again Fig. 5. That is:

$$\frac{r}{sin\left(\frac{\pi}{4}\right)} = \frac{d}{sin\left(\frac{\theta}{2}\right)}.$$

If only one of the vehicles turning left is longer than $\approx 0.7S$, only a *two-way synchronization* will be safe. That is, simultaneous left turns are only possible either on the south/north or east/west lanes, since trajectories do not coincide at any point in time.

Interleaving driving through and left turning vehicles allows for an overall throughput of 6 or 8 vehicles every 9 cycles, depending on whether a two-way or a four-way synchronization is possible on the left-turn lanes. Note that this is more efficient than driving through alone as depicted in Fig. 3, which allows for 4 vehicles every 6 cycles, i.e., 8 vehicles in 12 cycles.

Right and left turns (Fig. 6): Since right and left turns do not conflict with each other, the crossing maneuver can be synchronized with the highest throughput of 12 to 10 vehicles per 6 cycles. Here, right turns allow for 4 vehicles every 3 cycles. Left turns allow for 2 to 4 vehicles every 6 cycles. Again, this depends on whether a two-way or a four-way synchronization is possible on the left-turn lanes as depicted in Fig. 6.

Considering overlength vehicles: So far, we have considered that every vehicle fits entirely in one sector. However, if one vehicle is longer than S, it will introduce an overlength

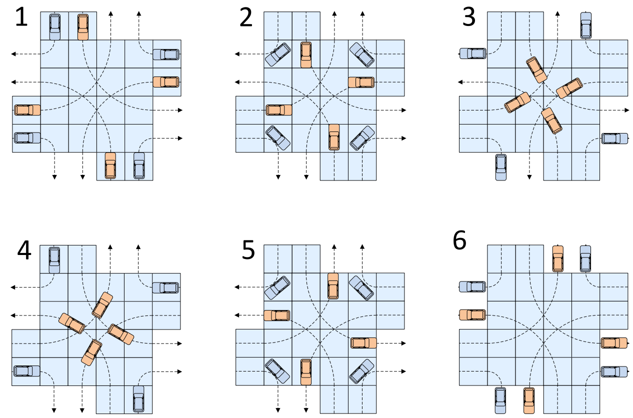

Fig. 6: Right and left turns are non-conflicting and combined result in the highest possible throughput. This leads to a throughput of 10 to 12 vehicles per 6 cycles. Similar to before, this figure visualizes four-way synchronization for left turns.

penalty that affects vehicle periods/inter-vehicle separations and, hence, also affects throughput. For $L > S$, we can compute the overlength penalty in terms of σ applying Eq. (1).

For example, having a vehicle with $L = 7.5m$ and a sector of $S = 5m$, this leads to $O = 3$ with $\sigma = 1m$, which corresponds to $0.6S$ or $3m$. Now, in the case of drive through only, the vehicle period becomes $6.6S$ instead of just $6S$ and throughput then reduces to 4 vehicles every 6.6 cycles. The same applies to the other crossing patterns as well.

V. The BRIP Protocol

BRIP (Ballroom Intersection Protocol) [1], has different types with different layouts as depicted in Fig. 7.

One of the core assumptions of BRIP is that the sector size has to be at least $L+W$, where L and W are the length and width of the largest vehicle crossing the intersection. Since the definition of a *cycle* is bound to the sector length, a *BRIP cycle* can be considerably larger than the previously discussed cycles in LTR.

BRIP Type I is a two-lane setup for through driving only. Concerning throughput, BRIP Type I can accommodate one vehicle per direction and BRIP cycle. Type II consists of a mix of through driving and right turns, similar to type IV, which combines left turns and right turns. These two

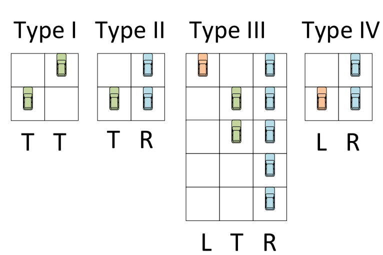

Fig. 7: Overview of arrival patterns for the different BRIP Types [1]. In this picture, T stands for through lane, whereas R and L stand for right- and left-turn lane respectively.

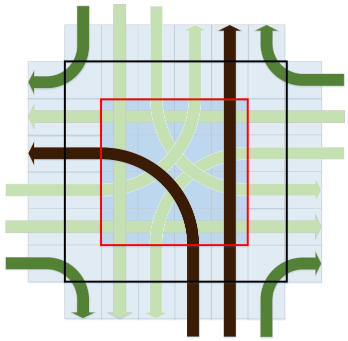

Fig. 8: Crossroad setup for BRIP Type III. Note that left-turn and drive-through maneuvers are confined to a two-lane space as marked by the red square. Right turns do not impact the remaining lanes.

types both lead to a throughput of 3 vehicles per direction every two BRIP cycles. Type III allows for 8×4 vehicles in the different directions every 5 BRIP cycles, however, considering a three-lane setup.

Concerning right turns: BRIP Types II, III and IV have a dedicated right-turn lane. All three types allow for one vehicle per BRIP cycle and direction to turn right. However, the remaining lanes in these three cases are completely independent of right turns, as depicted in Fig. 8 for the case of Type III. As a result, we separate the analysis of right turns from the rest of the lanes. We first perform a detailed comparison of our proposed LTR with BRIP Type I and Type III where right turns are not considered and then analyze the effect of a dedicated right-turn lane on the different protocols separately. Note that Type III is a generalization of Type II and Type IV. So Type II and IV are not explicitly included in this comparison.

VI. Evaluation

In this section, we compare the performance of LTR with BRIP Types I and III considering a constant speed of $30km/h$. Although this allows us to express throughput as vehicles per minute (instead of cycles), the speed value does not affect this comparison, nor the conclusion drawn from it. For Type III, we present both a comparison considering a dedicated right-turn lane as well as a two-lane version without right-turn lane.

Since in LTR the sector length S is independent of vehicles' lengths, we considered different variants in this comparison: LTR(3m) with $S = 3m$, LTR(5m) with $S = 5m$ and LTR(7m) with $S = 7m$. Recall that S in LTR is also equal to the width of a lane. As already described, BRIP's sector size is equal to $W + L$ (i.e., width plus length) of the longest vehicle possible.

We consider vehicle lengths between $3m$, which represents a rather short vehicle, and $30m$, which accounts for long trucks and public transportation vehicles like buses or trams [9].

Intersection size: Space requirements by all protocols involved are depicted in Fig. 9. Here, BRIP rapidly leads to substantially bigger intersection sizes as longer vehicles are allowed. This exceeds the space standard infrastructure usually provides. Due to the invariance of LTR with respect to vehicle lengths, the requirements towards the existing infrastructure are considerably less demanding. It should be noted that the values in Fig. 9 are representing BRIP Type I. For Type III, all values are 50% higher due to the need of 3 lanes instead of 2.

BRIP Type I: A comparison with Type I is depicted in Fig. 10. As it can be observed, for the case of only short vehicles, the throughput performance of BRIP is substantially higher than any of the LTR variants. However, with rising vehicle length, LTR outperforms BRIP increasingly. For example, for a length of around $7m$, LTR(3m) starts outperforming BRIP in the case that long vehicles are almost never present at the intersection (solid green line). If all vehicles at the intersection are the longest possible, LTR(3m) starts outperforming BRIP not until a maximum allowable length of $12m$ (dashed green line). Similar behavior can be seen for the other LTR variants as well.

BRIP Type III: A comparison with BRIP Type III is further shown in Fig. 11 where right turns are not considered and in Fig. 12 where a dedicated right-turn lane is considered.

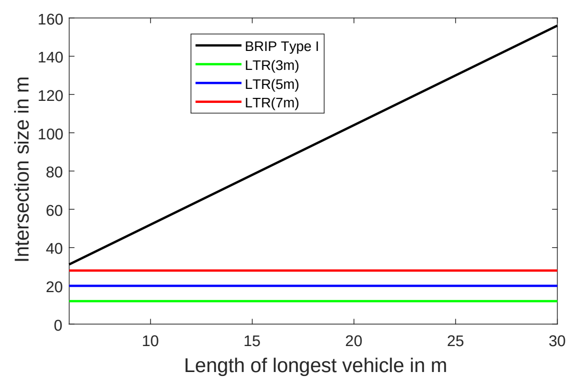

Fig. 9: Space requirements on the intersection with respect to vehicle length.

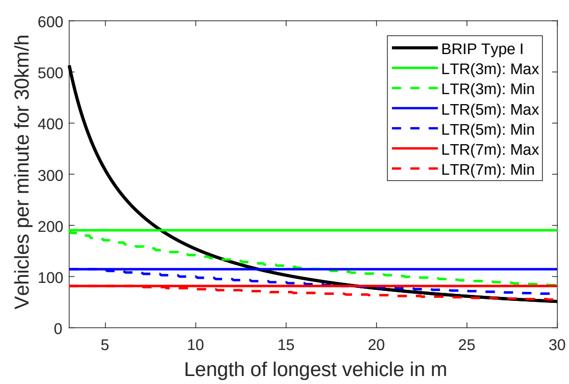

Fig. 10: Throughput compared to BRIP Type I and with respect vehicle length.

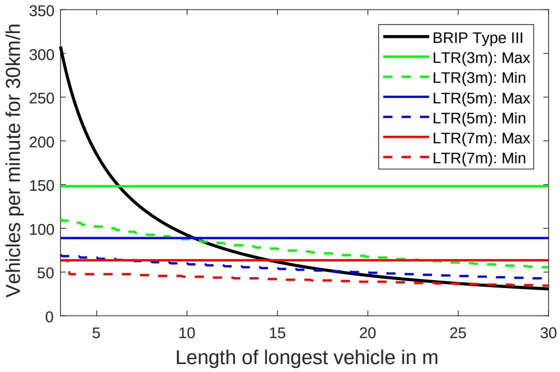

Fig. 11: Throughput compared to BRIP Type III with respect to vehicle length without considering right turns

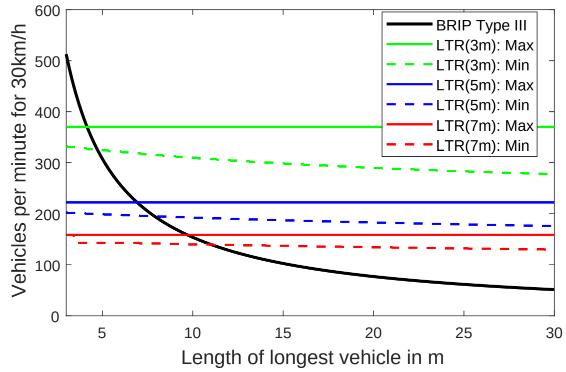

Fig. 12: Throughput compared to BRIP Type III with respect to vehicle length when considering right turns

When excluding right turns, LTR(3m), for example, starts having a higher throughput than BRIP for vehicle lengths that are greater than $7m$, provided that long vehicles are almost never present at the intersection (solid green line). If only long vehicles are present at the intersection, LTR(3m) does not start outperforming BRIP until a length of $11m$ (dashed green line). Again, a similar behavior can be observed by the other LTR variants.

When including right turns, the throughput shifts even more in favor of LTR, provided that this also has a dedicated right-turn lane preventing conflicts with drive-through vehicles. In this case, LTR(3m) starts outperforming BRIP for maximum vehicle lengths that are greater than $4m$ in the case, where long vehicles are almost never present at the intersection (solid green line). When only long vehicles are present at the intersection, LTR(3m) starts outperforming BRIP for a maximum vehicle length of $5m$ and greater (dashed green line). The other LTR variants behave similarly.

This result is due to the high frequency with which right turns can be performed (4 vehicles per 1 BRIP cycle and 3 LTR cycles respectively), where the effect of shorter cycles as of LTR ends up dominating.

To summarize, BRIP provides very high throughput numbers and reasonable space requirements on the intersection for short (e.g., less than $5m$) and homogeneous vehicles. However, with increasing vehicle lengths, BRIP's space requirements increase drastically. The proposed LTR can outperform BRIP slightly in terms of throughput for BRIP Type I, and quite considerably for BRIP Type III, while requiring substantially less space. It therefore represents a valid alternative for situations where the existing infrastructure cannot be modified or when overlength vehicles are allowed at the intersection, but have a low probability of occurrence.

VII. Concluding Remarks

In this paper, we proposed a space-efficient traffic protocol for intelligent crossroads called LTR (Left turn, Through, Right turn). LTR's main goal is to accommodate overlength vehicles while keeping intersections small and still allowing for high vehicle throughput.

To this end, based on their low probability of occurrence, LTR handles overlength vehicles as exceptions and not the rule. When an overlength vehicle is present at the intersection, the protocol introduces an overlength penalty adjusting inter-vehicle separations at the cost of a lower throughput.

We have compared the proposed LTR with an BRIP, an existing traffic protocol from the literature, showing that LTR has a better performance in terms of throughput and space requirements as overlength vehicles become longer.

As future work, we plan to analyze how traffic protocols are affected by communication loss taking, among others, probabilistic techniques into account.

Acknowledgment

This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - MA 6480/3-1.

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