Controlled Intra-Platoon Collisions for Emergency Braking in Close-Distance Driving Arrangements

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***Abstract*—The increasing degree of automation and communication makes it possible that vehicles travel at short separations of a few meters, i.e., in a close-distance driving arrangement or platoon. This leads to higher energy/fuel savings and an increased vehicle throughput on roads, among other benefits. Whereas a considerable amount of effort has been dedicated to cruise control in such settings, techniques for emergency braking have been paid less attention. However, this is of paramount importance for a safe operation in such settings and requires special attention. The goal is to reduce the overall stopping distance when braking in an emergency, while keeping a compact platoon, i.e., inter-vehicle separations as short as possible to maximize benefits. This turns out to be challenging, in particular, if vehicles have different braking capabilities, e.g., due to their type and/or loading conditions. In some cases, intra-platoon collisions may even be the only way to avoid major accidents. In this paper, we are concerned with this problem and propose an approach based on engineering *controlled* intra-platoon collisions. The idea is to minimize potential damage incurred by platoon vehicles, while reducing the overall stopping distance. We illustrate and evaluate our proposed approach for the case of a two-vehicle arrangement based on detailed simulations.**

I. Introduction

Road infrastructure worldwide cannot be extended at the same pace with which the number of vehicles grows. This has lead to increased congestion and thereby to considerable economic loss [1]. As a result, close-distance driving arrangements like platoons are attracting attention as a way to alleviate this situation — apart from further benefits such as energy/fuel savings [1] [2]. Platoons are possible thanks to increasing automation and communication between vehicles, which can be arranged to travel at short separations of $5\,\text{to}\,10m$ [3].

Up to date, most attention has been dedicated to designing and developing cruise controllers for platoons [4] [5] [6] [7] [8]. These controllers, apart from performing the longitudinal and lateral maneuvers, mainly focus on guaranteeing so-called *string stability* [9], where small variations in the separations between vehicles in the platoon's front are guaranteed not to amplify towards the rear.

On the other hand, ensuring string stability does not guarantee a safe braking [5] [10]. Even though cruise controllers regulate the speed of individual vehicles in a platoon and brake within a given range, they cannot handle situations like emergency braking. This is because all the platoon vehicles have to apply their maximum possible brake force in order to reduce the overall stopping distance of the platoon. Hence, the brake actuators work close to or at saturation. As a result, specialized brake controllers have to be designed for emergency braking.

Especially, the heterogeneous deceleration capabilities of vehicles in the platoon, for example, due to their type and/or loading conditions, need to be considered. If neglected, the deceleration magnitude achieved by one vehicle might not be achievable by its immediately following vehicles and an uncontrolled collision may occur.

To avoid intra-platoon collisions, one can either increase the separation between vehicles accordingly or force them all to brake as the worst or weakest vehicle in the platoon [11] [12]. While the first approach allows achieving the shortest possible stopping distance, it also results in the least compact, i.e., longest, platoons jeopardizing benefits. In contrast, the second approach allows for compact platoons, but it also incurs the longest possible stopping distance. As a consequence, trade-offs between these two approaches have been proposed recently [11] [12]. However, in some cases, avoiding collisions within a compact platoon may yield an overall stopping distance that is still insufficient to prevent collisions with traffic ahead.

In this paper, we are concerned with this problem and propose allowing for *controlled* intra-platoon collisions, i.e., where vehicles in the platoon incur almost none or insignificant damage, reducing the overall stopping distance to a great extent.

We consider a two-vehicle platoon with heterogeneous deceleration capabilities, i.e., different stopping distances, operating at an inter-vehicle separation of below $5m$. Further, we arrange vehicles such that the lead brakes at a higher deceleration rate than the trail vehicle and, hence, they collide at a given point in time. However, at the moment of collision, we ensure that the velocities of the two vehicles are (almost) the same. This leads to (very) low deformation forces and, hence, none or insignificant damage at vehicles.

Further, we show that the combined deceleration of the vehicle arrangement after collision is higher than that of the trail vehicle alone, i.e., the lead helps braking the trail vehicle. Thus, the resulting stopping distance is reduced considerably as we illustrate by detailed simulations.

Structure of the paper. Related work is presented in the next section, whereas Section III deals with the principles and fundamentals on which our controlled-collisions approach is based. In Section IV, we introduce our approach along with the full-fledged design scheme for the corresponding brake controllers. Section V is concerned with simulation results. Finally, Section VI concludes the paper.

II. Related Work

It has been shown that there is an increased likelihood of rear-end and sideswipe crashes regardless of the type of vehicles in a platoon [13]. In general, most of the existing approaches for braking in a platoon consider constant inter-vehicle separations and relatively compact platoons. For example, at separations below $5m$, the probability of inter-vehicle collisions and the relative velocities at impact were studied in [14], outlining the necessity of coordination among vehicles during braking.

A two-truck platoon was considered in [15] to study the impact of control system failures and the effects of driver reaction times on manual braking. It was shown that the trail vehicle has to brake at a higher deceleration magnitude than the lead to avoid collisions in such situations. Similarly, [10] proposed to have the better braking vehicle as the trail to ensure safety during braking. In this case, they show that the inter-vehicle separation can be as short as $2m$.

The benefits of communicating the lead vehicle's braking information through wireless messages to all vehicles was shown to enhance safety [16], when compared to just relying only on radar or information from neighboring vehicles.

The approaches in [17] demonstrate how to achieve synchronization between vehicles in a platoon using vehicle-to-vehicle (V2V) communication and ensure a safe braking even at inter-vehicle separations of $8m$. Similarly, [18] proposed a braking control protocol based on V2V communication.

In contrast to the above works, the approach presented in [12] [11] not only focuses on avoiding intra-platoon collisions, but also on reducing the overall stopping distance of compact platoons with inter-vehicle separations of below $5m$.

To the best of our knowledge, the proposed technique in this paper is the first to contemplate controlled intra-platoon collisions as a means to further reduce the overall stopping distance of compact platoons. This may seem to contradict responsibility-sensitive safety (RSS), where an automated vehicle drives in such a way that it avoids accidents and also compensates for the mistakes of other road users [19]. However, as mentioned before, avoiding intra-platoon collisions might result in high-velocity crashes with other road users. In such extreme situations, our proposed technique may be the only way to prevent accidents with traffic ahead.

III. Principles and Fundamentals

In this section, we discuss our assumptions and introduce concepts and principles on which our approach is based.

A. Assumptions

As mentioned above, we consider a two-vehicle platoon operating at inter-vehicle separations of below $5m$. Further, following assumptions are made:

- The two vehicles belong to the same category of passenger and/or utility vehicles, i.e., two-axle vehicles. Our approach can also be extended to multi-axle vehicles like trucks with corresponding changes in the design of their brake-by-wire controllers.
- Both vehicles know their respective maximum deceleration magnitudes. This implies that they are able to estimate/measure their loading conditions requiring the corresponding sensors to that end.
- Vehicles are equipped with brake-by-wire systems as these are suitable for automation and control, rather than conventional brake systems.
- Their brake-by-wire controllers can accurately track an assigned deceleration and a rate of change of deceleration (i.e., jerk) up to 3 decimal places and there are no quantization errors. Note that lifting this assumption, i.e., considering that brake-by-wire controllers are less accurate, yields longer stopping distances, but does not affect the validity of the proposed approach.
- Vehicles are equipped with an IEEE 802.11p based transceiver that is capable of broadcasting and receiving messages over the allocated frequency band in Europe. Further, we assume that the inter-vehicle communication is stable and there are no extreme situations like complete loss of communication.[1]
- Finally, before initiating an emergency brake maneuver, we assume a cruise speed of around $30m/s$ or $108km/h$, which is a typical highway speed. This can be increased at the expense of harder reliability requirements on communication.

B. Communication Strategy

Our communication strategy is based on the two kinds of IEEE 802.11p messages namely cooperative awareness message (CAM) [20] and decentralized environmental notification message (DENM) [21]. These are sent over a dedicated frequency band (in Europe allocated by the European Telecommunications Standards Institute (ETSI)).

CAM messages are used for periodic position updates from any vehicle to its surrounding vehicles, whereas DENM messages contain information about a road hazard or an abnormal traffic condition. In our work, the two vehicles in the platoon periodically broadcast their position, speed, and acceleration/deceleration values through CAM messages, and only the lead vehicle broadcasts a DENM message to initiate an emergency braking.

The only modification in our strategy is with respect to the transmission period of these messages. We deviate from the standard's recommendation of a $100ms$ transmission period especially for CAMs, and choose a period of $20ms$ for both these message types (to account for speeds of around

[1] On the other hand, to account for communication loss, one can increase inter-vehicle separations accordingly such that there is sufficient time to perform an emergency brake and thereby dissolve the platoon.

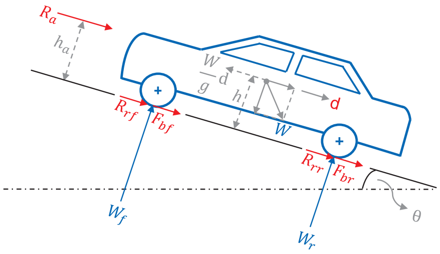

Fig. 1. Forces on a two-axle vehicle during braking [24]

$100 km/h$). Our choice is based on the observations by truck manufacturers as mentioned in [22].

In accordance with the field trials [23], we can neglect any propagation delay by these messages. This implies that any message broadcast is received instantaneously. On the other hand, we assume that there is a $20ms$ delay to process the message contents and initiate appropriate actions.

After broadcasting a DENM message to initiate emergency braking, the lead vehicle does not brake immediately, but with a $20ms$ delay. This delay ensures a synchronized braking of the two vehicles and has a negligible impact of at most $0.6m$ (i.e., $30m/s \cdot 0.02ms$) on the overall stopping distance.

C. Stopping Distance

Fig. 1 shows the forces acting on a two-axle vehicle during braking, resulting in a linear deceleration d (in m/s^2) [24]:

$$\frac{F_b + f_r W cos(\theta) + R_a \pm W sin(\theta)}{W} = \frac{d}{g}, \quad (1)$$

where the brake forces at the front and rear axles, F_{bf} and F_{br} respectively are combined into one resultant total force F_b (in N). The rolling resistances at the front and rear wheels, R_{rf} and R_{rr} respectively, are also combined into $f_r W cos(\theta)$ (in N), where f_r is the coefficient of rolling resistance, and θ is the road grade or inclination in degrees. The weights acting on the front and rear axles W_f and W_r constitute the total vehicle weight W (in N) acting at the vehicle's center of gravity situated at a height h (in m) from the road surface.

As shown in Fig. 1, the aerodynamic resistance R_a (in N) is acting at a height h_a (in m) from the road surface and aids braking. On the other hand, the grade resistance $W sin(\theta)$ (in N) aids braking in an uphill and opposes it in a downhill, hence, the $\pm$ signs respectively. Finally, g is the acceleration due to gravity in m/s^2.

Based on these forces, the stopping distance S (in m) from an initial velocity V (in m/s) can be computed as follows [24]:

$$S = \frac{\gamma_m W}{2 g C_A} \ln\left(1 + \frac{C_A V^2}{\eta_b \mu W + f_r W \cos(\theta) \pm W sin(\theta)}\right), \quad (2)$$

where γ_m is referred to as *equivalent mass factor* and has a value of $1.03 - 1.05$ for passenger vehicles. It indicates that the brake system has to decelerate a mass slightly greater than the vehicle's mass due to moment of inertia of the rotating components. The coefficient of road adhesion is denoted as μ, whereas $C_A = \frac{\rho}{2} C_D A_f$ and $\eta_b = \frac{(\frac{d}{g})}{\mu}$.

In these expressions, ρ is the air-mass density in kg/m^3, C_D is a vehicle's aerodynamic drag coefficient, and A_f is its frontal area (in m^2) along the direction of travel. During platooning, C_D's magnitude would be reduced depending on the inter-vehicle separation resulting in lesser aerodynamic resistance and, hence, energy/fuel savings. These savings are optimum when the inter-vehicle separations are in the range of 1 to $4m$ as even the lead vehicle experiences benefits. For details on the same see [1] [2].

The maximum achievable deceleration is limited by the coefficient of road adhesion (μ). On dry asphalt surfaces, this is 0.85g which reduces to around 0.2g on snowy surfaces. Hence, η_b denotes a vehicle's braking efficiency [24].

IV. Controlling Intra-Platoon Collisions

In this section, we design our brake-by-wire controllers such that two colliding vehicles have (almost) the same speed at the moment of impact. For the sake of exposition, we first disregard controller-related effects like settling time, steady-state error, etc.

Let us now denote by d_i^{max} the maximum deceleration magnitude of the lead vehicle i, whereas d_j^{max} represents the maximum deceleration magnitude of the trail vehicle j. Since we assume that a collision happens, $d_i^{max} > d_j^{max}$ must hold.

In order to equalize speeds at the moment of impact, whereas d_j^{max} remains constant, we propose to linearly vary the lead vehicle's deceleration as follows:[2]

$$d_i(t) = -d_i^{max} + \kappa_i \cdot t, \quad (3)$$

where κ_i is a slope of deceleration change that needs to be computed. Integrating (3) results in the expression of the lead vehicle's velocity over time t:

$$v_i(t) = \int d_i(t) dt = V_i - d_i^{max} \cdot t + \frac{\kappa_i \cdot t^2}{2}, \quad (4)$$

where V_i is an integration constant and equal to the lead vehicle's speed just before it begins to adapt its deceleration. Now, integrating a second time, we obtain the expression of trajectory:

$$s_i(t) = \iint d_i(t) dt^2 = S_i + V_i \cdot t - \frac{d_i^{max} \cdot t^2}{2} + \frac{\kappa_i \cdot t^3}{6}, \quad (5)$$

where again S_i is an integration constant and equal to the lead vehicle's position just before adapting its deceleration.

In the same way, we can obtain the expression of velocity and trajectory for the trail vehicle by integrating its constant maximum deceleration. This then leads to:

$$v_j(t) = \int -d_j^{max} dt = V_j - d_j^{max} \cdot t, \quad (6)$$

and

$$s_j(t) = \iint -d_j^{max} dt^2 = S_j + V_j \cdot t - \frac{d_j^{max} \cdot t^2}{2}. \quad (7)$$

[2]In principle, one can vary the lead's deceleration in a different way, e.g., using a nonlinear equation, However, that will also unnecessarily complicate all computations.

TABLE I
CONTROLLER-DESIGN SPECIFICATIONS FOR REFERENCE TRACKING

Property	Value	Description	Reason
Overshoot	0%	The magnitude of deceleration (expressed as a percentage) during the transient that exceeds the steady-state value.	Since the controller reaches the saturation brake force, designing for an overshoot $>0\%$ is impractical resulting in a nonlinear behavior that is difficult to deal with.
Settling time	$\leq 400ms$	The time required to achieve a deceleration that remains within $\pm 2\%$ of the reference.	Since no overshoot is required, a feasible controller needs a longer time to settle.
Steady-state error	$\approx 0\%$	The difference between reference and achieved deceleration in the steady state.	A non-negligible steady-state error accumulates over time leading to intra-platoon collisions at higher velocities.
Feedback delay	$20ms$	Delay incurred in the feedback loop.	The delay due to data processing by sensor and to communicate the same back to the controller.

To equalize speeds at collision, we enforce $v_i(t_{coll}) = v_j(t_{coll})$, i.e., we equate (4) and (6), where t_{coll} is the point in time at which the vehicles collide. That is:

$$V_i - d_i^{max} \cdot t_{coll} + \kappa_i \cdot \frac{t_{coll}^2}{2} = V_j - d_j^{max} \cdot t_{coll},$$

which can be solved for t_{coll}, obtaining:

$$t_{coll} = \frac{\Delta d + \sqrt{(\Delta d)^2 + 2 \cdot \kappa_i \cdot \Delta v}}{\kappa_i}, \tag{8}$$

where $\Delta v = V_j - V_i$, and $\Delta d = |d_i^{max}| - |d_j^{max}|$.

To be able to compute t_{coll}, we need to determine the value of κ_i. This can be done considering that the inter-vehicle separation becomes zero at the moment of impact, i.e., $s_i(t_{coll}) - s_j(t_{coll}) = 0$. Hence, equating the difference between (5) and (7) to zero and considering an initial inter-vehicle separation of Δs, i.e., $S_j = S_i - \Delta s$, we arrive to:

$$\frac{\kappa_i \cdot t_{coll}^3}{6} - \frac{\Delta d \cdot t_{coll}^2}{2} - \Delta v \cdot t_{coll} + \Delta s = 0. \tag{9}$$

Now, we can solve (9) to obtain three roots, representing possible values for t_{coll}. One of these roots results in a positive real-valued κ_i when equalized to (8). That value of κ_i is the one that we need to use in (3) to ensure that vehicles have the same speed at the moment of impact. Note that the expression for κ_i is quite complex and, hence, we do not show here. This can be at best obtained using a symbolic equation solver (such as that provided with Matlab).

A. Accounting for Controller Effects

Unfortunately, a brake-by-wire controller cannot follow changes in its deceleration instantaneously, i.e., it cannot perform instantaneous *jerk* tracking, rather it initially exhibits a transient behavior before it settles and adapts its deceleration as per κ_i. Due to this behavior, the vehicles collide before the computed t_{coll} and their velocities will not be the same at impact.

Therefore, we need an expression that characterizes the controller's behavior to determine the actual time of collision and velocities at impact. To that end, we first derive our vehicle model. Since rolling and aerodynamic resistances aid braking, we can neglect them (see Fig. 1) yielding a linear and time-invariant (LTI) system, for which we obtain the following state-space representation:[3]

$$\dot{x}_i = 0 \cdot x_i + \frac{1}{\gamma_m \cdot m_i} \cdot u_i + z_i, \tag{10}$$

and

$$y_i = 1 \cdot x_i, \tag{11}$$

where the only state is the vehicle i's velocity in m/s denoted by x_i. Similarly, its deceleration in m/s^2 is $\dot{x}_i$, its mass in kilograms (kg) is m_i, and its equivalent mass factor is again denoted as γ_m.

Note that (10) states Newton's second law, i.e., the deceleration is equal to the (input) brake force u_i divided by the mass times the equivalent mass factor plus the disturbance z_i (e.g., grade force, etc.).

Further, the brake-by-wire controller has to meet the performance specifications in Table I for reference tracking (i.e., a constant deceleration). For simplicity, we consider that the same controller is used for jerk tracking (i.e., a varying deceleration). However, a separate controller optimized for jerk tracking can also be used to improve performance. Note that any controller technique can be used. In this paper, we use the proportional integral derivative (PID) due to its ease of design. The corresponding gains can be obtained using standard methods such as Root Locus or Pole Placement [25] and, hence, we do not elaborate on this any further.

We can now obtain an expression that characterizes the controller's behavior for a given reference input. We first obtain the transfer functions $G_{p_i}(s)$ from the state-space model represented by (10) and (11) and $G_{c_i}(s)$, i.e., the transfer function of the controller used. Since the controller regulates deceleration rather than velocity, an accelerometer described by $H(s) = \frac{s}{s+1}$ needs to be added as depicted in

[3]The standard state-space representation of a LTI system is: $\dot{x}_i = Ax_i + Bu_i + z_i$ and $y_i = Cx_i + Du_i$, where A, B, C, and D are the system, input, output, and feed-forward matrices respectively, u_i is the input vector, x_i is the state vector, and z_i is the disturbance vector. Note that we have one-element vectors x_i, u_i, and z_i and as a result, matrices A, B, C, and D become scalars. Further, to be consistent with the standard representation, we explicitly make the output y_i equal to our only state x_i.

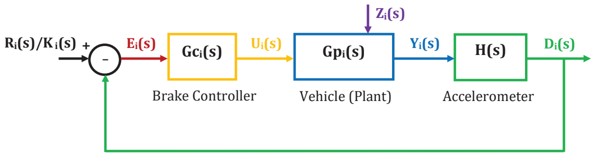

Fig. 2. Closed-loop control system

Fig. 2. Now assuming no disturbance, i.e., $Z_i(s) = 0$, we obtain the overall transfer function $G_i(s)$ as follows:

$$G_i(s) = \frac{Gc_i(s) \cdot Gp_i(s) \cdot H(s)}{1 + [Gc_i(s) \cdot Gp_i(s) \cdot H(s)]}. \tag{12}$$

After emergency braking is initiated, the lead vehicle's controller takes $400ms$ to settle at d_i^{max}, i.e., it performs reference tracking. We assume that, within the next $20ms$, κ_i's value is obtained solving (8) and (9) as mentioned above (i.e., under ideal conditions, ignoring controller-related effects). From $420ms$ onwards, the controller starts performing jerk tracking, where the lead vehicle's deceleration changes at the rate of κ_i. Assuming the controller takes another $400ms$ to settle and follow κ_i (which is the case as shown later), we must determine the deceleration, velocity, and position of the lead vehicle at $820ms$, i.e., after its controller settles again (this time, for jerk tracking).

Let us denote by $D_i(s)$ the lead vehicle's deceleration in the frequency domain s, which is a 5^{th} order transfer function obtained by multiplying the input to be tracked $K_i(s)$ by the transfer function $G_i(s)$. This transfer function can further be decomposed into partial fractions as follows:

$$D_i(s) = \frac{R_1}{s - p_1} + \frac{R_2}{s - p_2} + \frac{R_3}{s - p_3} + \frac{R_4}{s - p_4} + \frac{R_5}{s - p_5}, \tag{13}$$

where R_1 to R_5 are residues and p_1 to p_5 are poles. Applying the inverse Laplace transform, we obtain the lead vehicle's deceleration in the time domain t:

$$\begin{aligned} d_i(t) = &R_1 e^{p_1 t} + R_2 e^{p_2 t} + R_3 e^{p_3 t} \\ &+ R_4 e^{p_4 t} + R_5 e^{p_5 t} - d_i^{max}, \end{aligned} \tag{14}$$

where d_i^{max} is the initial deceleration value at $t = 0.42$ just before the controller switches from reference to jerk tracking. Now, integrating (14), we obtain the expression of velocity:

$$\begin{aligned} v_i(t) = &\frac{R_1 e^{p_1 t}}{p_1} + \frac{R_2 e^{p_2 t}}{p_2} + \frac{R_3 e^{p_3 t}}{p_3} + \frac{R_4 e^{p_4 t}}{p_4} \\ &+ \frac{R_5 e^{p_5 t}}{p_5} - d_i^{max} \cdot t + V_i, \end{aligned} \tag{15}$$

where V_i is the lead vehicle's velocity also at $t = 0.42$ (just before switching to jerk tracking). Finally, integrating (15) yields the trajectory:

$$\begin{aligned} s_i(t) = &\frac{R_1 e^{p_1 t}}{p_1^2} + \frac{R_2 e^{p_2 t}}{p_2^2} + \frac{R_3 e^{p_3 t}}{p_3^2} + \frac{R_4 e^{p_4 t}}{p_4^2} \\ &+ \frac{R_5 e^{p_5 t}}{p_5^2} - \frac{d_i^{max} \cdot t^2}{2} + V_i \cdot t + S_i, \end{aligned} \tag{16}$$

where again S_i is the lead vehicle's position at $t = 0.42$ (just before switching to jerk tracking).

Note that while we can reliably measure the lead vehicle's velocity V_{i0} and position S_{i0} after its controller initially settles at d_i^{max}, i.e., at $t = 0.4$, V_i and S_i, i.e., velocity and position at $t = 0.42$, need to be computed kinematically (akin to (6) and (7)). This is possible because the lead vehicle decelerates at a constant d_i^{max} from $t = 0.4$ to $t = 0.42$.

Now, the lead vehicle's controller starts jerk tracking at $t = 0.42$ and settles at $t = 0.82$. This duration of 0.4 is substituted as t along with d_i^{max}, V_i, and S_i in (14), (15), and (16), from which we obtain the deceleration, velocity, and position values respectively after the controller settles for jerk tracking at a rate of κ_i (which again was computed under ideal conditions, i.e., ignoring controller-related effects).

Now, these resulting values along with the corresponding values of the trail vehicle are used to recompute Δd, Δv, and Δs, i.e., the differences in the deceleration magnitudes, velocities, and positions of the two vehicles after the lead vehicle's controller settles for κ_i. Substituting these values together with (the previously obtained) κ_i in (9) allows computing the actual t_{coll}, i.e., taking controller-related effects into account.

Substituting this actual t_{coll} in (4) and replacing d_i^{max} and V_i with the values obtained from (14) and (15) respectively, we can determine the lead vehicle's velocity at impact.

An example. Consider a two-vehicle platoon cruising on a flat road ($\theta = 0$) at a speed of $30m/s$ and an initial inter-vehicle separation of $4m$. The lead vehicle has a mass $m_i = 3284kg$. Due to its loading conditions and considering $\gamma_m = 1.05$, its maximum deceleration rate $d_i^{max} = 7.28m/s^2$. Similarly for the trail vehicle, $m_j = 3265kg$ and $d_j^{max} = 4.76m/s^2$.

Based on Fig. 1, we modeled the vehicles along with their controllers in Matlab/Simulink. For the lead's controller, the integral gain K_{int} is 34482, whereas both the proportional and derivative gains are 0. Hence, $G_{c_i}(s) = \frac{34482}{s}$.

After $420ms$ of initiating emergency braking, $\Delta d = 2.511m/s^2$, $\Delta v = 0.806m/s$, $\Delta s = 3.905m$ and, hence, $\kappa_i = 2.287m/s^3$. Substituting these values in (8) yields $t_{coll} = 2.48s$, i.e., starting from the point in time of initiating emergency braking, the two vehicles collide at $2.90s$ $(0.42 + 2.48)$, ignoring controller-related effects.

For the decomposition as per (13), the residues are [0.228, 0, −0.228, 2.287, 0] and the poles are [−10, −1, 0, 0, 0]. For these residues, poles, and additionally considering that there are 3 poles at the same location, i.e., the origin in this case, (14) simplifies to:

$$d_i(t) = R_1 e^{p_1 t} + R_3 + R_4 \cdot t - d_i^{max}. \tag{17}$$

Integrating (17), we derive the corresponding expression of velocity as:

$$v_i(t) = \frac{R_1 e^{p_1 t}}{p_1} + R_3 \cdot t + \frac{R_4 \cdot t^2}{2} - d_i^{max} \cdot t + V_i. \tag{18}$$

Finally, integrating (18) yields the trajectory as:

$$s_i(t) = \frac{R_1 e^{p_1 t}}{p_1^2} + \frac{R_3 \cdot t^2}{2} + \frac{R_4 \cdot t^3}{6} - \frac{d_i^{max} \cdot t^2}{2} + V_i \cdot t + S_i. \tag{19}$$

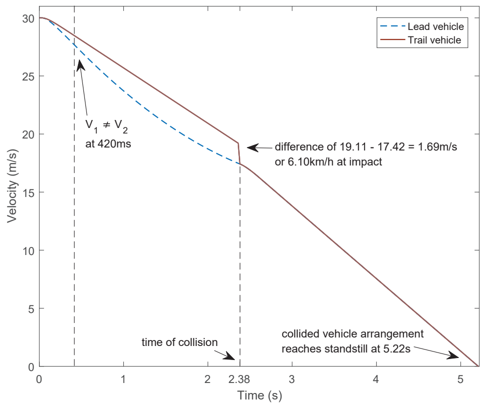

Fig. 3. Velocity profiles of the two vehicles in our simulation

At $420ms$, $V_i = 27.64m/s$ and $S_i = 19.20m$ (measured relative to the position at the moment of braking). Substituting $t = 0.4$ in (17), (18), and (19) respectively yields a lead vehicle's deceleration of $-6.527m/s^2$, velocity of $24.86m/s$, and position of $29.69m$, i.e., the values at time $820ms$. The values obtained by simulation are $-6.526m/s^2$, $24.85m/s$, and $29.69m$ respectively, which are actually very similar to that computed by our expressions.

We can further compute the trail vehicle's velocity and position at $t = 0.82$ kinematically obtaining $26.55m/s$ and $26.30m$ respectively. Hence, we have $\Delta d = 1.775m/s^2$, $\Delta v = 1.685m/s$, and $\Delta s = 3.398m$. Substituting these values along with κ_i in (9) yields $t_{coll} = 1.59$, i.e., the two vehicles are expected to collide at $2.41s$ $(0.82 + 1.59)$ due to controller-related effects at the lead vehicle.

Substituting $t = 1.59s$, and replacing V_i and d_i^{max} by $24.86m/s$ and $6.527m/s^2$ respectively in (4) yields a lead vehicle's velocity of $17.37m/s$ at impact. In the simulation, this is $17.42m/s$ as shown in Fig. 3. This minor difference is because the collision happens $3ms$ earlier than computed, i.e., at $2.38s$. Fig. 4 shows how the lead vehicle's deceleration changes along time. Even though the controller was designed for reference tracking it performs jerk tracking quite acceptably.

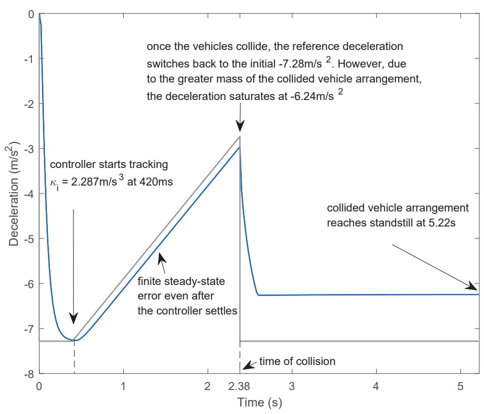

Fig. 4. Deceleration by lead vehicle in our simulation

The earlier collision can be reasoned by the fact that we have computed κ_i assuming ideal conditions, i.e., that the lead vehicle can instantaneously track any desired deceleration. On the other hand, we ignored aerodynamic and rolling resistance, which actually aid in braking the lead vehicle. As a result, it gets closer to the trail vehicle than computed by our expressions. Recall that our expressions were derived using the LTI model that accounts only for the brake force.

B. Minimizing the difference in velocity at impact

Minimizing the difference in velocity at the moment of collision allows reducing damage at vehicles. One possible way is to tune the controller to settle faster, in our case, at $200ms$, for jerk tracking. Note that we use the same controller. It is only the gains that are different for reference and jerk tracking. As a result, the previous $400ms$ settling time (as per Table I) results in PID gains that applies for reference tracking and another set of gains that settles the same controller in $200ms$ is used only for jerk tracking.

We determined the settling time value of $200ms$ by trial and error. Designing for a even shorter settling time produced oscillations and as a result, the controller took longer to settle. Note again that a separate controller optimized for jerk tracking can be used instead. However, for simplicity, we do not elaborate on this any further.

Even though the settling time can be reduced this way, it cannot be completely eliminated leading to a steady-state error even after the controller settles — see again Fig. 4. We can now quantify this steady-state error and adjust κ_i's value accordingly. To that end, let $E_s(s)$ denote the steady-state error in the frequency domain, which is determined as follows [25]:

$$E_s(s) = \lim_{s \to 0} s \cdot K_i(s)[1 - G_i(s)], \tag{20}$$

where again $K_i(s)$ is the input (deceleration) to be tracked and $G_i(s)$ is the transfer function as per (12).

An example. Consider the same 2-vehicle platoon as mentioned in the previous example. Due to the $200ms$ settling time, the lead vehicle's controller now uses $K_{int} = 68964$ for jerk tracking. Based on (12), we obtain $E_s(s) = \frac{K_i(s)}{20}$, i.e., the steady-state error is 5% of the input. Hence, the new $\kappa_i = 2.402m/s^3$ (i.e., $2.287 + (0.05 \cdot 2.287)$). With this value the difference in velocity at impact is reduced from $1.69m/s$ to $0.177m/s$ (i.e., $0.637km/h$). The vehicles now collide at $2.80s$, which is close to the previous value of $2.90s$ computed ignoring controller-related effects. This is because having a shorter settling time renders the controller more *ideal*.

Note that the error $E_s(s)$ is a function of the input jerk and not an absolute value. Therefore, it cannot be accounted for beforehand in our expressions for deceleration (14), velocity (15), and position (16).

V. Simulation Results

In this section, we evaluate our proposed controlled-collisions approach for emergency braking, in particular, we

TABLE II
VEHICLE DATA USED IN THE SIMULATION

Vehicle	m (in kg)	max. d (in g)	C_D	A_f (in m^2)	Controller gain K_{int}	Stopping Distance (in m)
Best	3284	0.7430	0.362	2.02	34482	67.78
Average	2367	0.5883	0.318	2.16	24853.5	83.96
Worst	3265	0.4864	0.325	2.02	34282.5	100.32

TABLE III
TOTAL PLATOON LENGTH FOR THE DIFFERENT APPROACHES

Platoon configuration	Least Stopping Distance (in m)	Least Platoon Length (in m)	Controlled Collisions (initial length in m)
Best & worst vehicle	42.54	11	11
Best & average vehicle	26.18	11	11
Average & worst vehicle	26.36	11	11

compare it with the approaches mentioned in the introduction namely *Least Platoon Length* and *Least Stopping Distance*. In the former approach, the lead vehicle brakes at the deceleration rate of the trail and as a result, the inter-vehicle separation is just $1m$. In the latter approach, the lead vehicle brakes at its maximum deceleration. Consequently, in order to ensure safety, the inter-vehicle separation must be at least the difference in stopping distance of the two vehicles.

A. Test Data

The vehicle data was randomly generated. We considered vehicle masses m in the range of $1000kg - 3500kg$ and the frontal areas A_f in the range of $2m^2 - 2.5m^2$, i.e., we consider passenger vehicles. Since the aerodynamic coefficients C_D of production cars are in the range of $0.311 - 0.475$ [24], the same was chosen.

We consider a dry asphalt surface and, hence, the coefficient of road adhesion μ is 0.85 [24]. This also implies that vehicles can achieve a maximum deceleration magnitude of 0.85g under optimal brake-force distribution to their axles. Hence, we chose the vehicles' maximum deceleration capabilities in the 0.5g $- 0.8$g range. Note that due to the equivalent mass factor γ_m, with a common value of 1.05, the corresponding maximum decelerations would be lesser.

Based on this data, we randomly generated 1000 vehicle data sets, from which we selected three. These are the vehicle as shown in Table II with the shortest stopping distance, henceforth referred to as *best* vehicle, the vehicle with the longest stopping distance, henceforth referred to as *worst* vehicle, and a vehicle with an average stopping distance, henceforth referred to as *average* vehicle. Note that we assume that every vehicle is $5m$ in length.

The stopping distances of vehicles when braking in isolation from a common initial velocity of $30m/s$ under their respective controller's action are shown in Table II (see rightmost column). Note that this also includes the $3m$ distance traveled due to a *dead time* in brake activation, i.e., brakes do not engaged immediately, but with a given delay (of typically $0.1s$).

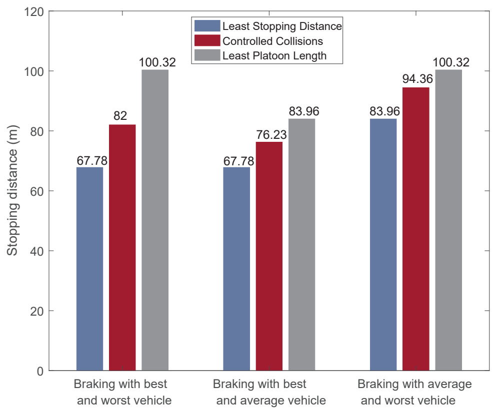

Fig. 5. Comparison of stopping distances with our controlled-collisions approach at $1m$ initial separation

B. Comparison of Stopping Distances

We simulated different 2-vehicle platoons with different combinations of vehicles from Table II and based on the model of Fig. 1 on a flat road ($\theta = 0$). Further, we arrange vehicles in each platoon such that the trail vehicle has a longer stopping distance than the lead.

Fig. 5 shows the resulting stopping distances by platoons when using the different approaches. In this experiment, we chose an inter-vehicle separation of $1m$ for both Least Platoon Length and our controlled-collisions approach at the moment of initiating braking. Least Stopping Distance requires a larger inter-vehicle separation to preserve safety as discussed next. Considering the platoon with the best and worst vehicle, our controlled-collisions approach allows for a stopping distance of $82m$, i.e., $18m$ shorter than Least Platoon Length.

On the other hand, the same platoon achieves a stopping distance of around $67m$ with the Least Stopping Distance approach. However, the inter-vehicle separation in this case is around $32m$ clearly affecting platooning benefits. With respect to the other platoons — best & average vehicle and average & worst vehicle — the inter-vehicle separation is around $16m$. The total platoon length for the approaches is present in Table III, whereas the time of collision along with the difference in velocity at impact with our proposed approach are given in Table IV.

Fig. 6 shows the resulting stopping distances for the same platoons. However, the initial separation between vehicles for Least Platoon Length and the proposed controlled-collisions

TABLE IV
SIMULATION RESULTS FOR $1m$ INITIAL SEPARATION

Platoon configuration	Time of collision (s)	Difference in velocity at impact (km/h)
Best & worst vehicle	1.18	2.94
Best & average vehicle	1.62	1.38
Average & worst vehicle	2.14	0.39

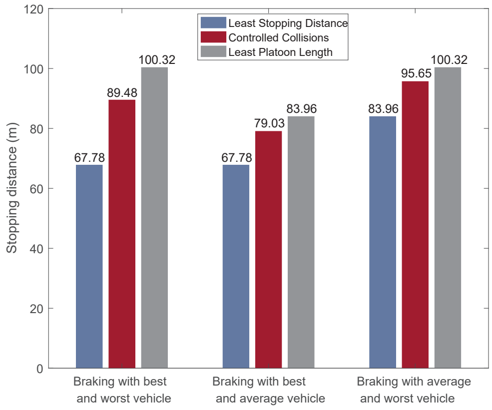

Fig. 6. Comparison of stopping distances with our controlled-collisions approach at $4m$ initial separation

approach is this time $4m$. As a result, the initial platoon length for our approach is now $14m$ instead of the previous $11m$.

It can be observed that the stopping distances with our approach are now longer than their respective counterparts at $1m$ initial separation. This is because the controlled collision happens at a later point in time due to the longer separation of $4m$. Still, our approach achieves shorter stopping distances in comparison to Least Platoon Length. The time of collision along with the difference in velocity at impact with our approach at $4m$ initial separation are present in Table V.

VI. Conclusions

In this work, we considered 2-vehicle platoons with heterogeneous braking capabilities operating at separations below $5m$ (for maximizing platoon benefits), particularly, $1m$ and $4m$. We analyzed the case of braking in an emergency and proposed an approach to engineer the collisions between vehicles with the aim of reducing the stopping distance. Further, we designed the corresponding brake-by-wire controllers. Our approach combines the braking capabilities of vehicles and achieves a shorter stopping distance in comparison to braking as the weaker trail vehicle.

As future work, we plan to extend our approach to platoons with more than 2 vehicles. In this case, the complexity increases considerably, since collisions between more than two vehicles in the platoon will have to be synchronized.

TABLE V
Simulation results for $4m$ initial separation

Platoon configuration	Time of collision (s)	Difference in velocity at impact (km/h)
Best & worst vehicle	2.80	0.63
Best & average vehicle	3.72	0.20
Average & worst vehicle	4.66	0.03

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